Debt Rescheduling with Multiple Lenders: Relying on the Information of Others

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Abstract

Can debt rescheduling decisions differ in a multiple lenders’ versus a single lender loan? Do multiple lenders efficiently react to information? We show that the precision of information plays an essential role. Foreclosing by one lender is disruptive so that a lender can rationally wait for the decision of other lenders, rescheduling her loan, if she expects that other lenders receive more precise information. We develop a Bayesian game where signals of different precision are randomly distributed to lenders. Both, premature liquidation and excessive rescheduling are possible in equilibrium, according to the pattern of information. However this is a second-best outcome, given that private information cannot be optimally shared.

Keywords: Overlending, debt contracts, insolvency, illiquidity, liquidation, relationship lending, multiple lenders, Bayesian games. JEL: G32, G33, D82, D86.

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1 Introduction

Large defaults by firms or sovereign debtors repeatedly raise the concern that lenders are deaf to alarm bells and do not act in time, letting the costs of the ensuing default inflate. Confirming these worries, CEOs at prominent banks have occasionally admitted to poor lending decisions.\footnote{To illustrate, “Lord Stevenson, the former chairman of HBOS, has admitted that the bank was guilty of “a lot of mistaken lending” in the run up to its near collapse in 2008”. In: Former HBOS chairman admits bank was guilty of ‘a lot of mistaken lending’, \textit{The Telegraph}, Monday December 10, 2012. Also, in the same article: “Lord Stevenson responded: I deeply regret the mistakes made in the corporate lending book, and with the wisdom of hindsight I wish we could have done things to obviate them.”} Often they share the responsibility with other lenders, since loans to large borrowers usually originate from multiple creditors — a feature that in many countries is also shared by smaller size loans, as witnessed by extensive empirical findings (Detragiache et al., 2000; Ongena and Smith, 2000; Berger et al., 2001). It is therefore natural to ask whether the presence of multiple lenders may be related to excessive lending and unwarranted roll-over of existing debt.

The passivity issue is treated in Diamond (2004), who bases his analysis on the idea that where creditors’ legal rights are poorly enforced there is little incentive for lenders to ask for repayment by going to court — because this reduces the overall value of collectibles. Diamond then argues that creditors’ multiplicity can create a commitment to stop refinancing a borrower if the signal received points to his misbehavior, provided the debt contract is made short term (i.e. shorter than the project’s cash-delivery time). Commitment comes from a Nash equilibrium at the refinancing stage where there is a run on the debtor’s assets. An increase in enforcement costs increases the tendency to passivity and the number of creditors needed to overcome it.

By contrast, the prominent view is that a bias towards liquidation prevails under multiple lenders, stemming either from an insufficient collateralizing of the debt, with lenders precipitating a “run” on the debtor’s assets, or...
from free-riding incentives in renegotiation which may hamper the recovery of valuable concerns (Bolton and Scharfstein, 1996; Gertner and Scharfstein, 1991; Detragiache and Garella, 1996). While the circumstances leading to excess liquidation or insufficient participation in debt restructuring are fairly well explored, the literature so far has not analyzed whether multiple lenders may also suffer from passivity, and why, delaying the necessary resolutions.

Empirically, a negative correlation has been observed between the number of lenders and the quality of borrowers; see Foglia et al. (1998) for Italy, Degryse and Ongena (2001) for Norway, Elsas and Krahnen (1998) and Harhoff and Korting (1998) for Germany. This may be taken as an indication that the premature liquidation bias is not universal, as it would imply a predominance of “type 1 errors” (worthwhile borrowers cut off) and not of type 2 (allowing bad loans to continue). In the literature one can find theories that, as well as our model, may accommodate the empirical negative relation: adverse selection bias at the stage when loans are granted, as in Bris and Welch (2005), with multiple lenders attracting “lemons”, or ineffective monitoring by lenders after granting the loan (Carletti, 2004).

The present paper explores the incentives to reschedule or foreclose when there are multiple lenders. In our analysis both excessive liquidation and excessive rescheduling are possible. Our novel result, however, is that multiple lenders may be inclined to reschedule when receiving information that would otherwise have triggered liquidation in a sole lender loan. The result is based on information asymmetries between lenders. We assume that the information available to individual creditors can differ in quality, i.e., it may be more or less accurate or precise. We show that a lender receiving a given piece of unfavorable information will often take a different decision depending on whether he is the sole lender or is part of a loan involving multiple lenders.

In everyday life, decisions are often delegated due to differences in the

\footnote{In Carletti, Cerasi and Daltung (2007), however, results on monitoring are nuanced.}
quality of information. For instance, when choosing a restaurant as tourists, we rely upon our own judgment if alone, while in company of locals we will rely upon them to pick the restaurant. Similarly, when voting in a committee we sometimes abstain and rely on colleagues with superior information (Feddersen and Pesendorfer, 1996). However, there are counterexamples, e.g. when personal preferences matter or when the ranking of alternatives is subjective, so that the precise payoff functions and rules of the game matter.

What then if a lender has to decide between going for liquidation or rescheduling a loan? If other lenders are also involved, she may wonder if her information is as reliable or accurate as that of the other lenders. We argue that, if it is likely that the others have superior information, then one may wish to avoid triggering liquidation and instead decide to reschedule. Specifically, we show that it is rational for a lender receiving unfavorable but imprecise information to be less inclined to foreclose than if he were the sole lender. Of course, given that information can be more or less precise, the opposite argument that comes to mind is whether a lender receiving imprecise but favorable information may also decide to rely on others to be better informed, with a rebalancing effect offsetting the tendency to more rescheduling. The point is that there is an asymmetry here: a lender with favorable information, be it precise or not, does not change her behavior because as sole lender she would also reschedule.

In our analysis lenders may be differentially informed because information is soft and therefore difficult to share. Two lenders participate in the financing of a project. The project can either be successful and repay the loans at maturity or it can run into problems, implying that rescheduling of both loans is needed if the project is to continue. At that stage, each creditor must decide between rescheduling or foreclosing after receiving signals about the continuation value of the project. The signals can be of two types defined by their quality. A high-quality signal is more informative than a low-quality one (as under an inclusion relation) and may point to the same
or to a different course of action. A lender observes her own signal and is aware of its quality, but does not know the quality and content of the signal received by the other lender. In the basic model, there is no exchange of information and the creditors do not coordinate their actions. In an extension we discuss why communication between lenders will not ensure complete information sharing.

We analyze the equilibria of the game of incomplete information played by lenders and show that the equilibria always entail more rescheduling by poorly informed lenders then under a sole lender arrangement. There are cases where premature liquidation can also arise in equilibrium. In particular, this occurs when a lender receives a poor quality signal that is very unfavorable to continuation and when the probability of the other lender being better informed is small enough. We also discuss the efficiency properties of the equilibria. The issue is whether there is too much rescheduling, i.e., too much “wait and see”, or by contrast too much premature liquidation. We find that, although decisions are inefficient compared to the first best under full sharing of information, they need not be inefficient in a second-best sense given the constraint that information cannot be shared. From a social point of view, the creditors’ equilibrium strategies may exhibit optimal reliance on the possibility that others are better informed, i.e., the strategies optimally trade-off the risk of unwarranted continuation against that of premature liquidation.

The literature on creditors’ passivity is not large. Existing explanations, besides the one by Diamond (2004), rely on the banks’ incentives to hide their problem loans (Rajan, 1994; Mitchell, 2001) or to “gamble for resurrection” (Perotti, 1993). Our explanation is therefore novel. With respect to the issue of coordination failures and liquidation bias, Gennaioli and Rossi (2012) discuss the use of floating charge debt and a dual class of debt. We take the short-term debt contract form as exogenous and stress instead the effects of information asymmetries on the lenders’ incentives to reschedule
loans. Carletti et al. (2007) discuss the possibility that monitoring incentives can in some cases be enhanced by the presence of multiple lenders; Dewatripont and Maskin (1995) analyze the basic externality in monitoring with two lenders. There is no monitoring activity in our model, i.e., information is taken to exogenously become available to the bank managers.

With respect to actions determined by the information of other players, the main strand of research is that of herding phenomena (Banerjee, 1992; Scharfstein and Stein, 1990). Herding has been tested as an explanation for excessive lending in Japan by Uchida and Nakagawa (2007) and in sovereign debtors crises (Calvo and Mendoza, 2000, and the references therein). However, herding implies that before taking his decision a player observes other players’ actions, in a sequence of moves, so as to infer something about these players’ information. Without that inference a player would act the same way as if she were alone. Our set-up, by contrast, hinges upon a player not being able to observe or to infer the type of information received by the other player. We show that the problem may arise whether or not the players’ actions are simultaneous; we discuss an extension to sequential moves where this applies. Hence, although herding also leads to decisions that differ from what a player would have done if alone, the mechanism of “reliance” we describe is different.

To interpret our results, it should be emphasized that they also hold when distortions are introduced in the payoffs that tip the balance towards more premature liquidation: in an extension, we show that the likelihood of triggering premature liquidation increases if an advantage to be the first creditor to go for liquidation is introduced. However, we also show the incentives to “rely” nevertheless persist and mitigates the effect of the first-mover advantage. When such an advantage is present, the comparison with a sole lender’s behavior is of course not literally possible and can only be made by analogy with the full information situation. Overall, our model rationalizes that postponement of action (or “passivity”) will be observed in loans with
multiple lenders and discusses some of the conditions leading to such behavior. To illustrate, if it were common knowledge that one particular lender is more likely to be better informed, then she will be the one to whom liquidation decisions are delegated (Franks and Sussman, 2005, discuss the empirical importance of concentrated liquidation rights).

The paper develops as follows. In Section 2 we present the model and derive some properties of the signals. In Section 3 we derive the equilibrium strategies at the rescheduling stage. In Section 4 we discuss the efficiency properties of the equilibria. In Section 5, we explore additional issues and extensions of the basic model. Section 6 concludes and discusses some empirical implications.

2 The model

The game unfolds over three periods. At date 0, an entrepreneur with no initial wealth seeks financing for a project. Two lenders, henceforth the banks, participate in equal measure to the provision of funds by means of debt contracts. The project is large or banks are small, so that financing must be obtained from two banks. The amount to be raised from each bank, \( L \), is normalized to 1. For each loan, the face value of the repayment due at date 1 is denoted by \( B \). The credit market is competitive and lenders earn zero expected profit in equilibrium. Since the project is of fixed size, the conditions leading to positive profits for banks, like in Bennardo et al. (2009) and Attar et al. (2010), do not apply here. To simplify the exposition, the opportunity cost of funds is set equal to zero.

With probability \( \gamma \), where \( 0 < \gamma < 1 \), the project succeeds (a good state of Nature realizes) and is completed by date 1, yielding the total return \( 2R \). We assume \( R \) large enough to ensure \( R \geq B \) for any face value \( B \) consistent with nonnegative expected profits to lenders — this implies that \( B > 1 \) will result, given the normalization \( L = 1 \). With probability \( 1 - \gamma \), the project
is not completed and runs into difficulties (bad state of Nature); the loan then cannot be paid back at date 1 as scheduled. A creditor then has two options. He can either foreclose on his loan or roll it over, namely allow the debtor to repay one period later than due. The project continues only if both banks reschedule, otherwise it is liquidated; i.e., foreclosing by one bank eventually forces foreclosing by the other lender.\textsuperscript{3}

In the bad state of Nature, each bank recuperates the amount $L < 1$ at date 1 if it forecloses. For instance, as means of realizing the project, the debtor produced or acquired assets that act as collateral and can be individually repossessed by the banks. If one bank seizes its share of these assets, continuation of the project is unfeasible, which is why foreclosure by one bank forces foreclosing by the other.\textsuperscript{4} In our context, rescheduling therefore amounts to a commitment to wait for repayment conditional on continuation of the project. If the project is allowed to continue, the assets in place are transformed into a random return available at date 2. This yields the amount $2X$ where the random variable $X$ has an absolutely continuous distribution with the interval $[0, 1]$ as support.\textsuperscript{5} Thus, the total final return upon continuation is always less than the amount borrowed. Because the banks own equal claims, they share the total final return equally, i.e., each bank recoups $X$. Table 1 summarizes the players' payoffs.

The prior (date 0) expected value of $X$ is $\mathbb{E}$. At date 1, however, before the rescheduling or foreclosing decision, each bank independently obtains some information about $X$. This information can be of two types: poor or good. A lender is accordingly said to be either poorly informed or well

\textsuperscript{3}The framework so far resembles Dewatripont and Maskin (1995).

\textsuperscript{4}Hart and Moore (1998) and von Thadden et al. (2010), among others, assume constant returns to scale: if, say, half the assets are seized, the project continues but at half size. By contrast, our project is indivisible or assets in place are perfect complements. Equivalently, seizure of half the assets is sufficiently detrimental to make continuation unambiguously unprofitable for the other lender.

\textsuperscript{5}Throughout we use capital letters to denote a random variable and the corresponding lower case letter for a particular realization.
informed (interchangeably, “better informed”). Poor information (e.g., “rumors”) is represented by the signal $Y$ which for simplicity both lenders are assumed to observe. Better information is the observation of $Y$ and of some additional signal $Z$. The probability that a lender observes both $Y$ and $Z$ is denoted $\theta$. With probability $1 - \theta$ a lender observes only $Y$. Whether well or poorly informed, a lender does not know if the other has observed $Z$, hence she does not know the “type” of the other lender.

**TABLE 1**

*Payoffs from the project*

<table>
<thead>
<tr>
<th></th>
<th>Good state</th>
<th>Bad state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>liquidation</td>
<td>continuation</td>
</tr>
<tr>
<td>Entrepreneur</td>
<td>$2(R - B)$</td>
<td>0</td>
</tr>
<tr>
<td>Banks</td>
<td>$2(B - 1)$</td>
<td>$2(L - 1)$</td>
</tr>
<tr>
<td>Total</td>
<td>$2(R - 1)$</td>
<td>$2(L - 1)$</td>
</tr>
</tbody>
</table>

The expected value of $X$ given $Y = y$ is denoted by $\pi(y)$, a strictly increasing function, i.e., a larger $y$ means more favorable information. The expected value of $X$ given $Y = y$ and $Z = z$ is denoted by $\pi(y, z)$, also an increasing function. Conditional on poor information, continuation is expected to be more profitable than liquidation if $\pi(y) \geq L$. Conditional on better information, it is expected to be more profitable if $\pi(y, z) \geq L$. The following assumption ensures that better information always matters.

**Assumption 1:** $\Pr(\pi(Y) < L)$ is positive. $\Pr(\pi(y, Z) > L \mid y)$ is positive for any realization $y$. 

8
The condition characterizes the information differential between the signals. A poorly informed lender expects continuation to be unprofitable when \( \bar{\pi}(y) < L \). Such values of \( y \) occur with positive probability. However, for any such \( y \), a poorly informed lender knows that there is a positive probability that he would change his mind if he were better informed. In other words, no matter how bad the rumors, the observation of the additional signal is valuable from a decision-making point of view.

**Assumption 2:** The random variables \( X, Y \) and \( Z \) are affiliated.

The assumption that the random variables are affiliated ensures that the conditional expectation \( E[g(Y, Z) \mid Y = y] \) is nondecreasing in \( y \), for any nondecreasing function \( g \).\(^6\)

The situation we have in mind is one where it can be part of an equilibrium for a poorly informed bank to disregard unfavorable information; that is, the bank may reschedule even though its expectations satisfy \( \bar{\pi}(y) < L \). The intuition is that a poorly informed lender may rely on the possibility that the other lender is better informed and is therefore able to make more appropriate decisions. If the other lender forecloses, then foreclosing will anyway be forced upon the poorly informed lender who chose to reschedule. For such a reliance strategy to be part of an equilibrium, “better” information must matter from a decision-making point of view, i.e., the event \( \bar{\pi}(y, Z) > L \) must have positive probability even though \( \bar{\pi}(y) < L \). This is the rationale for Assumption 1.

In the situation we consider, banks do not share their information and do not coordinate their actions. One reason is that the information received by the banks is nonverifiable or soft. Thus, any exchange of information at date 1 is cheap talk. In particular, it can be shown that announcements about the profitability of continuation are not credible if a small first-mover advantage

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\(^6\)See Milgrom and Weber (1982). If \( f(x, y, z) \) is the joint density, the random variables are affiliated when \( \ln f(x, y, z) \) has nonnegative cross derivatives.
to foreclosure is introduced in our setup. This possibility is analyzed in Section 5 which also discusses several extensions of our basic setup.

The assumption that continuation requires refinancing by both lenders needs comment. If external financing could be found, a project that runs into difficulties could be continued even after a refusal to reschedule by the original lenders. Also, it is possible that one of the original lender tries to buy the other lender out of the loan. The first case, issuance of new debt to be placed on the market, may be too costly because of “lemons” problems such as discussed in Detragiache et al. (2000). External investors would not know whether a refusal to refinance by the original lenders stems out of an efficient or inefficient decision. The other way out — that one of the original lender refinances the whole project — may be unfeasible for balance sheet reasons, as assumed here, or because of bargaining failures. Indeed, with many lenders, bargaining over lenders’ quotas will be costly and subject to hold-up problems. There is much evidence on the difficulty of agreements between creditors and the fact that the risk of bargaining failure is greater the larger the number of creditors (see Gilson et al., 1990; Asquith et al., 1994; and Gilson, 1997). In this respect, our example with two lenders must be taken only as illustrative. However, even in the case of two lenders, the well known bargaining failures due to asymmetric information will arise.

3 Rescheduling decisions

In the present Section we analyze the problem faced by lenders at date 1, when the bad state of Nature has occurred. Let for and res refer to foreclosing and rescheduling respectively. Strategies are denoted by $\alpha$, the probability that a bank plays for. We write $\alpha(y)$ for the strategy played by a bank which learns only $y$; similarly, $\alpha(y, z)$ is the strategy played by a better informed bank which learns both $y$ and $z$. A bank’s expected payoff is denoted by $u$. 
Bank $i$’s expected payoff from playing $for$ does not depend on bank $j$’s strategy, where $i, j = 1, 2$, and $i \neq j$. The payoff is also unaffected by the bank’s information. Thus,

$$u_i(for, \alpha_j | y) = u_i(for, \alpha_j | y, z) = L, \quad \text{for all } y, z. \quad (1)$$

This is not so for the expected payoff from playing $res$. If bank $i$ is well informed, its expected payoff conditional on observing $y$ and $z$ is

$$u_i(res, \alpha_j | y, z) = (1 - \theta) [\alpha_j(y)L + (1 - \alpha_j(y)) \pi(y, z)] + \theta [\alpha_j(y, z)L + (1 - \alpha_j(y, z)) \pi(y, z)]. \quad (2)$$

On the right-hand side, $\theta$ is the probability that bank $j$ is well informed. If the two lenders had different probabilities of being well informed, $\theta_i \neq \theta_j$, the right-hand side would contain $\theta_j$ in lieu of $\theta$. Bank $i$’s payoff depends on bank $j$’s strategies, which depend on the information available to that bank. When bank $j$ forecloses, foreclosing is forced upon bank $i$ as well, hence the first term in $L$ in the two bracketed expressions. When bank $j$ also reschedules, the project is allowed to continue and bank $i$’s payoff is the expected value of continuation conditional on the available information.

If bank $i$ is poorly informed, the expected payoff from $res$ is

$$u_i(res, \alpha_j | y) = (1 - \theta) [\alpha_j(y)L + (1 - \alpha_j(y)) \pi(y)] + \theta E[\alpha_j(y, Z)L + (1 - \alpha_j(y, Z)) \pi(y, Z) | y]. \quad (3)$$

The expression is similar to (2) except for the expectation operator in (3) because a poorly informed bank does not observe $Z$.

Comparing (1) and (2), it is clear that $for$ is a weakly dominated strategy for a well informed bank if $\pi(y, z) > L$. Rescheduling then yields a larger expected return than foreclosing if the other bank also reschedules with some
probability. Conversely, res is weakly dominated if $\pi(y, z) < L$. When the equality holds there is a tie; we take it that the bank then reschedules. By elimination of weakly dominated strategy, therefore, a well informed bank chooses

$$\alpha^*(y, z) = \begin{cases} 
1 & \text{if } \pi(y, z) < L, \\
0 & \text{if } \pi(y, z) \geq L.
\end{cases} \quad (4)$$

This coincides with the socially optimal decision conditional on the observation of $(y, z)$. It is also the decision rule that a well informed lender would follow in a sole lender arrangement.

To derive the best response of a poorly informed bank, observe that in the bad state of Nature $\max(\pi(y, z), L)$ is the expected return that would accrue to each bank under the socially efficient rescheduling decision. For a poorly informed bank, given the observation of $y$, the expected value of this quantity is $E[\max(\pi(y, Z), L) \mid y]$. Using the equilibrium strategy (4) and substituting in (3), the expected payoff from res for a poorly informed bank can therefore be rewritten as

$$u_i(res, \alpha_j \mid y) = (1 - \theta) [\alpha_j(y)L + (1 - \alpha_j(y)) \pi(y)] + \theta E[\max(\pi(y, Z), L) \mid y]. \quad (5)$$

The best response of a poorly informed bank depends on its own expectations about the value of continuation, on the other bank’s strategy when the latter is also poorly informed, $\alpha_j(y)$, and on the likelihood that the other bank is better informed, $\theta$. When a poorly informed bank’s expectations satisfy $\pi(y) \geq L$, it is obviously better to play res, as would also be done in a sole lender arrangement. When $\pi(y) < L$, the bank’s best option depends on how pessimistic it is about the value of continuation and on the probability that the other bank is better informed; the expression in (5) is then strictly increasing in $\theta$. If $\theta$ is sufficiently large, say close to unity, it is best to play res and rely on the other bank to make the appropriate decision. Conversely, if $\theta$ is close to zero, the best move is to play for. Moreover,
when \( x(y) < L \), the expression in (5) is increasing in \( \alpha_j(y) \). The greater the probability that the other bank forecloses when poorly informed, the safer it is to reschedule when one is also poorly informed because the probability of a “wrong” continuation is then smaller.

**Proposition 1** In equilibrium, well informed banks follow the socially optimal decisions rule as in (4). Poorly informed banks reschedule their loan when \( \bar{x}(y) \geq L \). When \( \bar{x}(y) < L \), poorly informed banks reschedule if

\[
\theta \geq \bar{\theta}(y) \equiv \frac{L - \bar{x}(y)}{E[\max(\bar{x}(y, Z), L) | y] - \bar{x}(y)}.
\]

For \( \bar{x}(y) < L \) and \( \theta < \bar{\theta}(y) \), there are two possible pairs of equilibrium strategies:

(i) In the symmetric equilibrium \( M \), poorly informed banks play a mixed strategy, rescheduling with the probability

\[
1 - \alpha^*(y) = \frac{\theta}{1 - \theta} \left( \frac{E[\max(\bar{x}(y, Z), L) | y] - L}{L - \bar{x}(y)} \right).
\]

(ii) In the asymmetric equilibrium \( P \), poorly informed banks play pure strategies: one bank reschedules, the other forecloses.

**Proof.** See the Appendix.

The essence of the result is that a bank with unfavorable but imprecise information may reschedule because it seems better to delegate decision making to the other lender. From the perspective of a poorly informed bank, the other lender may have received more precise information. When this is sufficiently likely, a poorly informed bank completely disregards its own information and relies fully on the possibility that the other bank is better informed. Temporizing through a rescheduling decision is then a dominant strategy; see the proof.

When the likelihood of the other bank being well informed is small, there are two possibilities. In the pure strategy equilibrium \( P \), one bank is passive
when poorly informed: it always reschedules and therefore “delegates” to the other lender the liquidation versus continuation decision. In turn, the lender “in charge” forecloses, thereby forcing liquidation, when its information is poor and sufficiently unfavorable, discounting the possibility that the other bank may have obtained superior information that favors continuation. In the symmetric strategy equilibrium $M$, poorly informed banks are indifferent between rescheduling or foreclosing when the information is poor and sufficiently unfavorable. The greater the likelihood that the other bank is well informed, the larger the probability of rescheduling, i.e., of relying on the other lender’s decision.

It is worth emphasizing that if probabilities of being informed differed across lenders, $\theta_i \neq \theta_j$, the expression for $i$’s equilibrium mixed strategy would display $\theta_j$ rather than $\theta$.

It is useful to present the equilibrium strategies under poor information explicitly as a function of the lender’s information, given the exogenous parameter $\theta$. Let $X_p = \pi(Y)$ denote the expected return from continuation under poor information, considered as a random variable. The subscript $p$ means “poor”. A particular realization is denoted by $x_p$. Similarly the expected return from continuation under better information is the random variable $X_b = \pi(Y, Z)$, where the subscript $b$ means “better”. Because $\pi(y)$ is strictly increasing, the conditional distribution of $X_b$ given $Y = y$ can be written as a function of $x_p$. Thus, the critical $\tilde{\theta}$ defined in Proposition 1 can be rewritten as

$$\tilde{\theta}(x_p) \equiv \frac{L - x_p}{E[\max(X_b, L) | x_p] - x_p}.$$  

(8)

**Lemma 1** Let $x_p^\text{min}$ be the worst possible expectation when information is poor. Then $\tilde{\theta}(x_p)$ is strictly decreasing over the interval $[x_p^\text{min}, L]$ with $1 > \tilde{\theta}(x_p^\text{min}) > \tilde{\theta}(L) = 0$.

**Proof.** See the Appendix.
The lemma follows from the Assumptions 1 and 2. Because the function \( \hat{\theta}(\bar{x}_p) \) is strictly decreasing over the interval \([\bar{x}_p^{\min}, L] \), it has an inverse over the interval \([0, \theta_c] \) where \( \theta_c \equiv \hat{\theta}(\bar{x}_p^{\min}) \). Denote this inverse by \( \varphi(\theta) \). Obviously \( \varphi(\theta) \leq L \) with strict inequality when \( \theta > 0 \). Let us now define

\[
\hat{x}(\theta) \equiv \begin{cases} 
\bar{x}_p^{\min} & \text{if } \theta \in [\theta_c, 1], \\
\varphi(\theta) & \text{if } \theta \in [0, \theta_c). 
\end{cases}
\]

The function is represented in Figure 1. Note that the curve \( \hat{x}(\theta) \) coincides with the vertical axis when \( \theta \geq \theta_c \). Our results can now be reformulated as follows.

**Corollary 1** For any \( \theta \), poorly informed lenders reschedule with probability equal to unity when \( x_p \geq \hat{x}(\theta) \). For \( \theta < \theta_c \), there are two equilibria which differ with respect to the actions of poorly informed lenders when \( \bar{x}_p < \hat{x}(\theta) \): either (i) they then play the mixed strategy prescribed by equilibrium M or (ii) they play the pure strategies prescribed by equilibrium P, i.e., one lender forecloses while the other reschedules.

![Fig. 1 Strategies under poor information](image)
The corollary makes explicit the difference with the decisions that would be taken in a single-lender loan. Consider the regions in Figure 1. By definition, \( MP = \{ (\bar{x}_p, \theta) : \bar{x}_p < \hat{x}(\theta) \} \), \( R_1 = \{ (\bar{x}_p, \theta) : \hat{x}(\theta) \leq \bar{x}_p < L \} \), and \( R_2 = \{ (\bar{x}_p, \theta) : \bar{x}_p \geq L \} \). When poorly informed, the single lender would reschedule only if \( \bar{x}_p \geq L \), i.e., he would reschedule in the region \( R_2 \) and foreclose in either \( MP \) or \( R_1 \). By contrast, in a two-lender arrangement, poorly informed lenders reschedule as a pure strategy in either \( R_1 \) or \( R_2 \). In the region \( MP \), there are two possibilities depending on the equilibrium: (i) either poorly informed lenders reschedule with the probabilities defined in (7); or (ii) one of the lender always forecloses while the other always reschedules. Altogether, for any positive \( \theta \), a lender in a two-lender arrangement forecloses less often than he would if he were the sole lender. Moreover, for any positive \( \theta \), there will be cases (in every equilibrium) where the project is refinanced even though both lenders have obtained information that would have triggered liquidation in a single-lender arrangement.

4 Inefficiencies

Compared with the first-best under shared information, inefficient continuation or inefficient liquidation is not surprising. Still, it is of interest to explore the nature and extent of the inefficiency. Moreover, while the perfect sharing of information represents an obvious benchmark, it may be that improvements could be achieved even though information is not shared. For instance, bank managers may tighten the rescheduling policy as a response to regulatory pressure, to reduce the overall risk of the bank portfolio, as when prudential regulation is reinforced. In our model, this could imply that a poorly informed bank behaves as in a sole lender arrangement.

Let \( \overline{v} \) denote the amount recuperated on average by each lender from a project that runs into difficulties; \( \overline{v} < 1 \) because the amount recuperated is either \( L \) or \( X \). Recall that a project is successful with probability \( \gamma \), in
which case it yields the return $2R$. The net expected value of a project is therefore
\[ \gamma 2R + (1 - \gamma)2\overline{v} - 2. \]
Inefficient continuation or liquidation reduce $\overline{v}$ and therefore reduce the net expected value of projects.\(^7\)

**Comparison with the first best.** To start, we characterize the first best when information is shared. Written as a function of $\theta$, the expected amount recuperated from unsuccessful projects is
\[ \overline{v}_{FB}(\theta) = (1 - \theta)^2 E \max (X_p, L) + \left( 1 - (1 - \theta)^2 \right) E \max (X_b, L). \quad (9) \]
The probability that both banks are poorly informed is $(1 - \theta)^2$. When information is shared, the banks know that only poor information is available and reschedule if $X_p \geq L$. This explains the first term in (9). With the complementary probability at least one bank is well informed. Because information is shared, the banks now reschedule if $X_b \geq L$, which yields the second term. Note that $E \max (X_b, L) > E \max (X_p, L)$ because $X_b$ is more informative than $X_p$ with respect to the value of continuation.\(^8\) Obviously, $\overline{v}_{FB}(\theta)$ is increasing in $\theta$.

Consider now the case where information is not shared, given that each bank plays the socially optimal — and equilibrium — strategy when well informed. As noted in the preceding section, the strategy of a poorly informed bank can be expressed as a function of $X_p$. We write the strategies as $\alpha_1(X_p)$ and $\alpha_2(X_p)$ for bank 1 and bank 2 respectively. The amount that is expected to be recuperated by each lender from unsuccessful projects is

\[ \text{\footnotesize \(^7\)Because the banks earn zero expected profits, each loan has face value $B$ satisfying $\gamma B + (1 - \gamma)\overline{v} = 1$. The larger $\overline{v}$, the smaller $B$ or equivalently the smaller the contractual rate of interest.}

\[ \text{\footnotesize \(^8\)The inequality follows from Jensen’s inequality because $\max(x, L)$ is a convex function and $E(X_b \mid X_p) = X_p$.} \]
then
\[
\overline{v}(\theta) = \theta^2 E \max(X_b, L) \\
+ \sum_{i=1}^{2} (1 - \theta) \theta E \{ \alpha_i(X_p) L + (1 - \alpha_i(X_p)) E[\max(X_b, L) \mid X_p] \} \\
+ (1 - \theta)^2 E \left[ L + (1 - \alpha_1(X_p))(1 - \alpha_2(X_p))(X_p - L) \right].
\] (10)

The first term is for the case where both lenders are well informed, the second for the case where one is well informed and the other poorly informed. In the third term, both lenders are poorly informed. If at least one forecloses, both banks get \( L \); if both reschedule, they each get the expected value of continuation conditional on the available information.

Substituting for the equilibrium strategies in equation (10) yields the equilibrium outcome which we denote by \( \overline{v}^*(\theta) \). Compared with the first best under perfect sharing of information, the inefficiency or welfare loss is
\[
\Delta(\theta) = \overline{v}_{FB}(\theta) - \overline{v}^*(\theta).
\]

This expression is positive if there is inefficient rescheduling or inefficient liquidation. In equilibrium, given the strategies described in the preceding section, inefficient rescheduling can only arise if both lenders are poorly informed. If both banks are well informed there is no inefficient decision. If both are poorly informed and \( \overline{\pi}_p < L \), the efficient outcome would be \( L \) for each bank; however, with probability \( (1 - \alpha_1^*(\overline{\pi}_p))(1 - \alpha_2^*(\overline{\pi}_p)) \) the banks reschedule and each obtains \( \overline{\pi}_p \) instead of \( L \). If bank 1 is informed and bank 2 is not (or the converse), the first best is \( \max(\overline{\pi}_b, L) \) but the poorly informed bank may inefficiently trigger liquidation. It is easily verified that
\[
\Delta(\theta) = (1 - \theta)^2 E[(1 - \alpha_1^*(X_p))(1 - \alpha_2^*(X_p)) \max(L - X_p, 0)] \\
+ (1 - \theta) \theta E \{(\alpha_1^*(X_p) + \alpha_2^*(X_p)) E[\max(X_b - L, 0) \mid X_p]\}. 
\] (11)

The first term in the right-hand side is for the case where both lenders are poorly informed, the second term for the case where one is poorly informed and the other well informed.
When $\theta \geq \theta_c$, the second term vanishes because both banks always reschedule when poorly informed. The welfare loss then reduces to

$$\Delta(\theta) = (1 - \theta)^2 E \max(L - \bar{X}_p, 0).$$

(12)

and is solely due to inefficient continuation. This occurs in the region $R_1$ of Figure 1.

When $\theta < \theta_c$, the first term in (11) remains positive but the second term is now positive as well. Inefficient continuation occurs in the region $R_1$. When $\bar{X}_p < \bar{x}(\theta)$, a poorly informed lender randomizes between foreclosure and rescheduling in the mixed strategy equilibrium. Hence, there is then both inefficient continuation and inefficient liquidation in the region $MP$ of Figure 1. In the asymmetric pure strategy equilibrium, when $\bar{X}_p < \bar{x}(\theta)$ one bank always forecloses if poorly informed; the other bank always reschedules if poorly informed. In the region $MP$ there is now inefficient liquidation but no inefficient continuation. The results are summarized in the following proposition.

**Proposition 2** Compared with the first-best decisions under shared information, there is inefficient rescheduling if $\theta \in [\theta_c, 1)$; there is both inefficient rescheduling and inefficient liquidation if $\theta \in (0, \theta_c)$.

When $\theta \geq \theta_c$, poorly informed banks always reschedule. Rescheduling may therefore occur even though both banks have unfavorable information. Compared with the first best, the problem is then too much “creditor passivity”. When $\theta < \theta_c$, inefficient liquidation also occurs. The bank “in charge” may foreclose even though the other lender is well informed and has obtained favorable information. In the mixed strategy equilibrium, a poorly informed bank may also inefficiently liquidate.

Finally, note that the second term in (11) vanishes as $\theta$ goes to zero. The first term also vanishes because lenders then foreclose whenever $\bar{X}_p < L$. In other words, there is maximum waste of information when the probability
that individual banks are well informed is neither too large nor too small. When the information is on average either very good ($\theta$ close to unity) or very bad ($\theta$ close to zero), the social loss from the non-sharing of information is negligible. Relying on the other bank to be well informed has negligible social cost if indeed the other bank is very likely to be well informed. Conversely, when the likelihood is small, at least one bank will almost be certain to liquidate — this is the bank “in charge” in equilibrium $P$ or both banks in the symmetric equilibrium.

**Second-best decision rules.** We now inquire whether the outcome can be improved by imposing decision rules on banks, subject to the constraint that the rules are consistent with the banks’ private information. Second-best optimal decision rules potentially differ from the equilibrium strategies ones only in the event that banks are poorly informed. To characterize the second-best rules, we therefore choose $\alpha_1$ and $\alpha_2$ in equation (10) so as to maximize $\bar{v}(\theta)$.

**Proposition 3** Subject to the constraint that lenders cannot share information, the following decision rules for poorly informed banks are second-best optimal: if $\theta \geq \theta_c$, the banks should always reschedule; if $\theta < \theta_c$, one bank should always reschedule while the other should reschedule if $\bar{x}_p \geq \bar{x}(\theta)$ and otherwise foreclose.

**Proof.** See the Appendix.

The result is surprising. When lenders obtain information that is not shared and if the likelihood of better information is sufficiently large (i.e., $\theta \geq \theta_c$), there is indeed excessive rescheduling in equilibrium compared with the first best under the perfect sharing of information. However, given the constraint that information cannot be shared, the non-cooperative equilibrium strategies are then socially optimal in a second-best sense. From a social point of view, in equilibrium each lender efficiently relies on the possibility that the other lender is better informed. In particular, the outcome
would be worse if poorly informed banks acted myopically, foreclosing when \( \bar{\pi}_p < L \). There would then be no inefficient rescheduling compared with the first best, but this would be more than compensated by too much inefficient liquidation. When the likelihood of better information is small (i.e., \( \theta < \theta_c \)), the equilibrium strategies are not necessarily second-best. In the region \( MP \) of Figure 1, the mixed strategy equilibrium entails inefficient continuation in a second-best sense. However, this is not so in the asymmetric pure strategy equilibrium. When poorly informed, the bank "in charge" efficiently trades-off the risk of inefficient continuation against the risk of inefficient liquidation.

5 Extensions

There are several ways in which the basic setup can be extended. We discuss the implications for the reliance strategy described in the preceding sections.

**Information.** The model made a sharp distinction between the poorly versus well informed status. A poorly informed lender knew that the other lender could not be less well informed; a well informed lender knew that the other could not be better informed. Moreover, there was perfect correlation between the information available to poorly informed lenders — they observe the same \( y \). Similarly, there was perfect correlation between the information available to the well informed — they observe the same \((y, z)\). These assumptions are made for simplicity and are not essential. In a more realistic environment, it is a matter of degree whether a lender is well or poorly informed.

To illustrate, suppose that the information is the observation of a signal within the set \( \{S_1, ..., S_K\} \) where the signal \( S_k \) is more informative than \( S_{k-1} \) with respect to the value of continuation, \( k = 2, ..., K \). For each lender, there is a positive probability of observing any one of these signals. In the model of the previous sections, \( K = 2 \) with \( S_1 \equiv Y \) and \( S_2 \equiv (Y, Z) \). When
$K > 2$, a lender observing $S_k = s_k$ with $k \notin \{1, K\}$ does not know whether the other lender is more or less precisely informed than herself. Nevertheless, our basic argument would remain the same. When a lender receives unfavorable information about the value of continuation (i.e., $\pi(s_k) < L$), whether he reschedules or forecloses depends on how pessimistic he is and on the likelihood that the other lender is better informed. The smaller $k$, the more likely it is that the other lender is better informed. Under appropriate conditions on the informational differential between signals, an equilibrium will be characterized by critical values $\widehat{x}_k \leq L$, with strict inequalities for small values of $k$, such that a lender observing $s_k$ reschedules if $\pi(s_k) \geq \widehat{x}_k$. The critical values $\widehat{x}_k$ are nondecreasing in $k$; that is, the rather poorly informed are more likely to reschedule when they have pessimistic expectations, the rather well informed are more likely to foreclose.

**Several lenders.** We now revert to the case of two signals as in the basic setup but allow the number of lenders to be $N + 1$ where $N > 1$. As before, foreclosure by a single lender triggers liquidation of the project. Denote by $\psi$ the probability that an individual lender is well informed. With minor modifications, the equilibrium will then be as before but with $\theta \equiv 1 - (1 - \psi)^N$, i.e., $\theta$ is the probability that at least one of the other $N$ lenders is well informed.

When $\theta \geq \theta_c$, poorly informed lenders reschedule irrespective of their information. Again, this is second-best efficient. When $\theta < \theta_c$, there is a symmetric equilibrium with lenders rescheduling if $\pi_p \geq \widehat{x}(\theta)$, where the latter function is defined as before; if $\pi_p < \widehat{x}(\theta)$, a poorly informed lender forecloses with the probability $\alpha^*(\pi_p)$ satisfying

$$
(1 - \alpha^*(\pi_p))^N = \frac{\theta}{1 - \theta} \left( \frac{E[\max(X_{k, \ell}, L) | \pi_p] - \pi_p}{L - \pi_p} \right).
$$

The argument is the same as in Proposition 1. From the point of view of a poorly informed lender observing $\pi_p < \widehat{x}(\theta)$, the left-hand side of (13)
is the probability that all other lenders reschedule when poorly informed.

Hence it is the probability of continuation if he himself reschedules and all
other lenders are poorly informed. When \( \theta < \theta_c \), there is also an asymmetric equilibrium in pure strategies: \( N \) lenders always reschedule irrespective of their information, while one lender, the one “in charge”, forecloses if \( \pi_p < \pi(\theta) \) and otherwise reschedules. Again, the asymmetric equilibrium is second-best efficient.

As \( N \) is increased with \( \psi \) constant, the likelihood that at least one of the other lenders is well informed increases. Loosely speaking, there is then more rescheduling but the welfare loss due to the non sharing of information nevertheless decreases.\(^9\) It is also of interest to consider the effect of a larger \( N \) while the information available to lenders as a group does not change, i.e., \( \psi \) is reduced so as to keep \( \theta \) constant. The consequence is now that the overall extent of rescheduling remains unchanged. In equation (13), the probability of foreclosing \( \alpha^*(\pi_p) \) is smaller so as to keep constant the probability of simultaneous rescheduling by \( N \) poorly informed lenders.

Of course, as \( N \) gets large, it becomes less reasonable to assume that foreclosure by a single lender triggers liquidation of the project. But then our basic argument still holds \( \text{mutatis mutandis} \) if \( \theta \) is redefined as the probability that at least \( N_1 \) of the other lenders are well informed, where foreclosure by \( N_1 \) lenders is sufficient to trigger liquidation.

**Relationship lending.** An interesting comparison is with the case where only one bank, say bank A, can observe both \( Z \) and \( Y \) with probability \( \theta_A > 0 \), while bank B can only observe \( Y \). The regions in Figure-1 above where rescheduling occurs then differ across banks. When poorly informed, bank A will reschedule only if \( \bar{x}_p \geq L \); otherwise it forecloses because it cannot rely upon bank B to be better informed. Obviously, when it is well

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\(^9\)For any given \( \psi \), \( \theta \) increases with \( N \). A sufficiently large \( N \) therefore implies \( \theta > \theta_c \), in which case poorly informed lenders always reschedule. The welfare loss then satisfies (12) and becomes arbitrarily small as \( N \) gets large.
informed, bank A takes the efficient action. The best reply of bank B when it is poorly informed is therefore to always reschedule: if bank A is also poorly informed, they both have the same information and bank A will foreclose anyway when $\bar{x}_p < L$; otherwise bank A has superior information and can be counted upon to take a more efficient decision. The equilibrium $P$ arises naturally here. Our model therefore suggests that in situations where the superior information of a lender is a structural feature of the loan arrangement, as when there is a relationship lender in the agreement, it is normal that this lender takes up the role of “leader”. The role would be preserved when there is a unique relationship bank and several other participants in a loan instead of just one. Syndicated loans can also be a case in point.

**First-mover advantage in foreclosure.** Returning to the two-lender case, we now explore the implications of a first-mover advantage in foreclosure. Assets in place at date 1 have a liquidation value of $2L$. If a bank forecloses while the other reschedules, the former gets to liquidate the amount $L + \varepsilon$. Foreclosure is then ultimately forced upon the other lender as well, but this lender is now second in line and can recuperate only the residual value $L - \varepsilon$. When both simultaneously foreclose, they each get $L$ as before.$^{10}$

A first-mover advantage in foreclosure tilts the equilibrium strategies towards more frequent foreclosure. As a result, it may remedy some of the excess rescheduling (compared with the first best), but at the cost of more frequent inefficient liquidation. To see this, let $\tilde{x}_{be}$ denote the equilibrium cutoff such that a well informed bank reschedules when its expectation

---

$^{10}$See von Thadden et al. (2010) for a similar formulation of the uncoordinated debt collection game. An interpretation is that the debt contracts give each lender a foreclosure right equal to $L + \varepsilon$, so that the sum of foreclosure rights exceeds assets in place. See also Gennaioli and Rossi (2012) who discuss the role of asymmetric debt structures for mitigating liquidation bias.
satisfies $\bar{\pi}_b \geq \bar{\pi}_{bc}$. When $\varepsilon$ is zero, the cutoff equals $L$ and decisions by well informed lenders are socially efficient. With a small positive $\varepsilon$, a well informed bank will forecloses when $\bar{\pi}_b$ is slightly above $L$. One reason is the possibility of gaining the first-mover advantage should the other lender reschedule. Another is that rescheduling is dangerous because the other lender might foreclose. A poorly informed bank will anticipate that the other lender, if well informed, will be more prone to foreclose. Its strategy may therefore also change and lean more towards foreclosure. At one extreme, if $\varepsilon$ is large enough, the advantage from foreclosing is so great that it becomes a dominant strategy, irrespective of the lender’s information.

Otherwise, when $\varepsilon$ is not too large, equilibrium strategies are similar to those already studied, though with more inefficient liquidation than if $\varepsilon = 0$. We illustrate with the case where $\theta$ is large enough for the strategy of a poorly informed bank to be unaffected by the first-mover advantage in foreclosure; that is, as before, poorly informed lenders reschedule irrespective of their information. Let $F(\cdot \mid \bar{\pi}_p)$ be the conditional cumulative distribution of $\bar{X}_b$ given $\bar{\pi}_p$. In the basic setup, a poorly informed lender reschedules irrespective of his information when $\theta$ is greater than

$$\hat{\theta}_c \equiv \frac{L - \bar{\pi}_p^{\min}}{E[\max(\bar{X}_b, L) \mid \bar{\pi}_p^{\min}] - \bar{\pi}_p^{\min}}$$

$$= \frac{L - \bar{\pi}_p^{\min} - \bar{\pi}_p^{\min}}{F(L \mid \bar{\pi}_p) L + (1 - F(L \mid \bar{\pi}_p)) E[\bar{X}_b \mid \bar{\pi}_p^{\min} \geq L, \bar{\pi}_p^{\min}] - \bar{\pi}_p^{\min}}.$$

With a positive $\varepsilon$, a poorly informed always reschedules when $\theta$ is greater than

$$\hat{\theta}_{ce} \equiv \frac{L + \varepsilon - \bar{\pi}_p^{\min}}{F(L + \varepsilon \mid \bar{\pi}_p) L + (1 - F(L + \varepsilon \mid \bar{\pi}_p)) E[\bar{X}_b \mid \bar{\pi}_p^{\min} \geq L + \varepsilon, \bar{\pi}_p^{\min}] - \bar{\pi}_p^{\min}}.$$

The argument for deriving the critical $\hat{\theta}_{ce}$ is as follows. First, if the poorly informed always reschedule, the equilibrium strategy of a well informed is to reschedule if $\bar{\pi}_b \geq L + \varepsilon$ and otherwise to foreclose. Second, given this
strategy on the part of the well informed, a lender observing $\pi_p$ and expecting the other lender to reschedule if poorly informed will himself reschedule if

$$(1 - \theta) \pi_p$$

$$+ \theta \{ F(L + \varepsilon | \pi_p)(L - \varepsilon) + (1 - F(L + \varepsilon | \pi_p)) E[\chi_b | \chi_b \geq L + \varepsilon, \pi_p] \}$$

$$\geq (1 - \theta)(L + \varepsilon) + \theta \{ F(L + \varepsilon | \pi_p)L + (1 - F(L + \varepsilon | \pi_p))(L + \varepsilon) \}.$$  

The left-hand side is the expected payoff from rescheduling, the right-hand side the payoff from foreclosure. It can be verified that, if the condition holds for some $\pi_p$ and $\theta$, it also holds for any larger value (indeed, given Assumption 2, $F(\cdot | \pi_p)$ is decreasing in $\pi_p$). This is easily seen to yield the critical $\bar{\theta}_{\text{ce}}$. This threshold is less than unity only if $\varepsilon$ is not too large. Our basic setup was the limiting case when $\varepsilon$ is negligible.

**Timing.** Abstracting from any first-mover advantage, we now consider the possibility that the loans do not have the same maturity. The consequence is that one bank will need to move first. Suppose that repayment of the loan from bank 1 is due at date 1, while that from bank 2 is due at the slightly later date 1b. As before, it is known at date 1 whether the project has run into difficulties and the lenders then independently receive information about the value of continuation.

The difference with the basic setup is that the lenders now play in sequence. Bank 1 makes its rescheduling versus foreclosure decision at date 1. Denote its strategy by $\alpha_1(\cdot)$ where the dot refers to the bank’s private information. Bank 2 makes its decision at date 1b after observing the action of bank 1. Its strategy is described by $\alpha_2(a_1, \cdot)$ where $a_1 \in \{\text{res, for}\}$ refers to bank 1’s action and the dot refers to bank 2’s private information. One can show the following. Let $\alpha_1^*(\cdot)$ and $\alpha_2^*(\cdot)$ be equilibrium strategies of the simultaneous game, as derived in the preceding sections. Then $\alpha_1(\cdot) = \alpha_1^*(\cdot)$ and $\alpha_2(\text{for,} \cdot) = 1, \alpha_2(\text{res,} \cdot) = \alpha_2^*(\cdot)$ are equilibrium strategies of the sequential game. In other words, with respect to continuation or liquidation
of the project, the outcome is the same as in the simultaneous game.

We provide the argument for the case where \( \theta \geq \theta_c \). In the simultaneous game, poorly informed lenders then always reschedule, i.e., \( \alpha_1^*(\xi_p) = \alpha_2^*(\xi_p) = 0 \) for all \( \xi_p \). In the sequential game, suppose the strategy of bank 1 is \( \alpha_1(\cdot) = \alpha_1^*(\cdot) \). When bank 1 has foreclosed, bank 2 cannot prevent liquidation of the project, so it is a best response for bank 2 to foreclose as well.\(^{11}\) When bank 1 has rescheduled, bank 2’s decision matters. If bank 2 is well informed, it is obviously best to foreclose if \( \xi_b < L \) and otherwise to reschedule, as prescribed by the equilibrium strategy of the simultaneous game. If bank 2 is poorly informed, it must infer from bank 1’s rescheduling decision that either bank 1 is also poorly informed or that it is well informed and \( \bar{X}_b \geq L \). If it has observed \( \xi_p \), bank 2’s expected payoff from rescheduling is therefore

\[
(1 - \theta) \xi_p + \theta E[\bar{X}_b | \bar{X}_b \geq L, \xi_p],
\]

Now

\[
(1 - \theta) \xi_p + \theta E[\bar{X}_b | \bar{X}_b \geq L, \xi_p] > (1 - \theta) \xi_p + \theta E[\max(\bar{X}_b, L) | \xi_p],
\]

where the right-hand side is at least as large as \( L \). By definition of \( \theta_c \), the above inequality holds for any \( \theta \geq \theta_c \) and any \( \xi_p \). Hence, bank 2’s best response when poorly informed is to reschedule when bank 1 has itself rescheduled.

Finally, consider bank 1’s decision given that bank 2 plays as stated. If bank 1 is well informed, it obviously takes the socially efficient decision. If it is poorly informed and reschedules, its expected payoff is

\[
(1 - \theta) \xi_p + \theta E[\max(\bar{X}_b, L) | \xi_p] \geq L.
\]

Hence bank 1 always reschedules when \( \theta \geq \theta_c \). Thus, for either bank, there

\(^{11}\)In this equilibrium, as will become obvious, a poorly informed bank 2 will infer from bank 1’s foreclosure decision that bank 1 was well informed and \( \bar{X}_b < L \).
is the same degree of reliance as in the simultaneous setup on the possibility that the other lender is better informed.\textsuperscript{12}

**Information sharing.** When the lenders move in sequence, some information is conveyed by the first mover’s action. We now consider explicit attempts to exchange information. Suppose the same situation as in our basic setup. However, at date 1, before they simultaneously decide on foreclosure or rescheduling, the lenders can now make announcements. For clarity, the communication game is taken to be played at date 1, while final decisions are made at the slightly later date 1b.

Consider the following possible announcements: “yes” and “no comment”. The announcement “yes” is shorthand for “I obtained favorable information and intend to reschedule”; “no comment” means that the lender says nothing. This is all that is needed to convey the information required for efficient decision making at date 1b. The following strategies are part of an equilibrium: at date 1, a lender says “yes” if he obtained favorable information (i.e., he is well informed and $\bar{x}_b \geq L$ or poorly informed and $\bar{x}_p \geq L$), he says “no comment” if he obtained unfavorable information; at date 1b, a lender who said “yes” reschedules as announced, a well informed lender who said “no comment” forecloses, a poorly informed lender who said “no comment” reschedules if the other lender said “yes” and otherwise forecloses. Indeed, following the announcement of “yes” by his counterpart, a poorly informed lender with unfavorable information infers that the other is well informed and that $\bar{x}_b \geq L$, hence he reschedules. Following “no comment”, he infers that either the other lender is well informed and $\bar{x}_b < L$ or that the other lender is also poorly informed, hence he forecloses. There is no incentive to mislead. In particular, it is in the interest of a well informed lender with favorable information to prevent foreclosure by the other

\textsuperscript{12}The behavior of bank 2, which disregards privately obtained “bad news”, is reminiscent of herding behavior as in Banerjee (1992). However, the analogy is misleading. Bank 1, which moves first, also disregards bad news when poorly informed.
lender (should he be poorly informed), hence to announce “yes”. Thus, a communication stage allows to coordinate on the first-best decisions at date.

It is easily seen, however, that such an equilibrium is not robust to a small first-mover advantage in foreclosure. A well informed bank announcing “yes” will play as announced only if $\pi_b \geq L + \varepsilon$. But then a well informed bank observing $\pi_b < L + \varepsilon$ would gain from making the same announcement if it thought it would be believed (which requires that the other lender is poorly informed), because it would then be the first mover in foreclosure. A similarly argument holds for a poorly informed lender observing $\pi_p < L + \varepsilon$. Thus, in equilibrium, favorable announcements will not be believed and will be equivalent to being told nothing, no matter how small $\varepsilon$. In the terminology of cheap-talk games, announcements are then neither self-committing nor self-signalling, hence they are not credible (see Farrell and Rabin, 1996). It follows that decisions at date 1b will be the same as in our basic setup.\footnote{The outcome is the same if the set of possible announcements is enriched to, say, “I am well informed and intend to reschedule”, “I am well informed and intend to foreclose”, and “I am poorly informed”.}

We now consider a variant that does not rely on cheap talk. As a result, the communication game is robust to a small first-mover advantage in foreclosure.\footnote{We thank a referee for pointing out this possibility.} A date 1b is added: at date 1 a lender can now take each of the three actions: foreclose, reschedule, and “wait”. This change opens up the possibility for lenders to “choose to be first movers”; specifically, a well informed bank may want to move first to guarantee that the efficient decision is taken. A lender who chose "wait" can still foreclose or reschedule at date 1b. Waiting expresses uneasiness with the situation. When the first-mover advantage in foreclosure is sufficiently small, it is easily shown that the following strategies are part of an equilibrium: at date 1, a lender reschedules if he has favorable information (i.e., he is well informed and $\pi_b \geq L + \varepsilon$ or poorly informed and $\pi_p \geq L + \varepsilon$), he forecloses if well informed and
<L + \varepsilon, otherwise he waits; at date 1b, a lender who waited reschedules if the other lender rescheduled at date 1 and otherwise forecloses. In this equilibrium, when both lenders played “wait”, they learn that they are both poorly informed and therefore coordinate on the socially efficient decisions when \varepsilon is small. When \varepsilon is zero, the play of “wait” by a poorly informed lender observing \bar{\pi}_p < L yields the expected payoff

\[(1 - \theta)L + \theta E[\max(\bar{X}_b, L) \mid \bar{\pi}_p] > L.\]  

Because the inequality is strict, the strategies described above constitute an equilibrium when \varepsilon is positive and sufficiently small. Note that in (14) the payoff from waiting exhibits reliance on the possibility that the other lender is better informed.

However, the model with two types of signal has been used only for convenience and is aimed at representing more general situations. It is sufficient to have \(K > 2\) signal types to jeopardize the feasibility of efficiently sharing information. For instance, suppose that the signals independently made available to the two lenders are drawn from the set \(\{S_1, ..., S_K\}\), as earlier in this section. For simplicity assume \(K = 3\) and let the probabilities be \(\theta_i\), for \(i = 1, 2, 3\). The actions foreclose, reschedule and wait will then not reveal all relevant information. Under appropriate conditions on the informational differential between signals and setting \(\varepsilon\) equal to zero, one may generate situations (with \(\theta_3\) high enough) where the equilibrium by which the maximum separation of types obtains is only semi-pooling. The following strategies are part of the equilibrium: at date 1, if a lender receives \(S_3\) she plays \textit{for} or \textit{res} immediately, according to what is efficient. The posteriors on observing a player playing \textit{for} or \textit{res} are therefore that she has received signal \(S_3\) with probability 1. However, a lender receiving \(S_2\) or \(S_1\) will play wait in order not to be pooled with type \(S_3\), least she should trigger the wrong action at stage 1b. This leads again to the situation we described with the two-signal model analyzed above. Hence, a reliance strategy similar
to the ones described in this paper will be part of the equilibrium.\textsuperscript{15}

Alternatively, one may stick to the simple two-signal framework but allow the more informative signal to be made available at either date 1 or date 1b but now with date 1b acting as a deadline, in the sense that a project cannot balance between liquidation or continuation beyond that date.\textsuperscript{16} Suppose both lenders waited because they had unfavorable information at date 1 but were poorly informed. At date 1b, a lender who remains poorly informed will take into account the possibility that by then the other lender may have obtained superior information that is favorable. Because they cannot wait any longer, the lenders are then essentially in the same situation as in our basic setup.

6 Concluding remarks

The idea explored in the present paper is that ex-post inefficient rescheduling may arise in the sense that lenders chose to disregard bad signals. We find however that, with some qualifications, ex-ante efficiency may be preserved in the sense that no better decision could have been taken on the basis of the information available to each lender. While our analysis helps rationalize the hindsight wisdom often shown by analysts and banks’ managers discussed at the outset, it also questions the view that a liquidation bias dominates at the refinancing stage under arrangements involving multiple lenders. An ill informed bank knows that its mistake in rescheduling can be corrected by better informed lenders, while a mistake in liquidating is, in our story,

\textsuperscript{15}As suggested by a referee, an extended communication game over as many stages as there are types of signals could restore the possibility of information sharing. With $K$ possible signals ordered in terms of informativeness and $K$ stages, a player observing the signal $S_k$ would announce his decision to reschedule or foreclose at stage $K - k$.

\textsuperscript{16}If no decision is reached by that date, assets in place are transformed into continuation and liquidation value falls to zero. Alternatively, continuation value is jeopardized and waiting amounts to liquidation.
irreversible.

In practice, foreclosing is a drastic decision, leading to a halt in the execution of a project and to liquidation, only if creditor rights are well protected and repossession of the debtor’s assets is swift and frictionless. This is not necessarily so. Prevailing codes ensure that debtor rights are preserved under liquidation or that debtors can appeal to special protection such as Chapter 11 in the U.S. We point out, however, that the “liquidation outcome” in our model need not mean bankruptcy in the legal sense. A loan’s liquidation value should be interpreted as reflecting the payment expected by a lender, taking into account the disruption caused by foreclosure, and given the collateral arrangements, the prevailing legislation, and the efficiency of the legal system.

Some empirical implications follow from our model. One should observe a decrease in liquidation by small lenders when a main bank is present in the loan. Such a pattern prevails when there is a relationship lender or main bank in a loan arrangement with multiple creditors. Overall, therefore, the small-lenders initiated liquidations should decrease when a relationship bank can be identified (syndicated loans may also fall in this category). A situation with asymmetries in the precision of information across lenders may also arise when lenders do not know whether the management of the borrower has disclosed the same information to all creditors or only to those with no conflict of interests (for instance only to those who do not lend to rivals so as to avoid disclosure of R&D results, as discussed in Bhattacharya and Chiesa, 1995; Yosha, 1995; Von Rheinbaben and Ruckes, 2004).\(^\text{17}\) In sectors with high R&D expenditures and where property rights on innovation are crucial, multiple lenders can exhibit the reliance mechanism more clearly than in mature and traditional ones. Also, if the first-mover advantage in liquidation is reduced or eliminated by the bankruptcy code, then our results

\(^{17}\text{Guiso and Minetti (2010) also allow for differential information disclosure to multiple lenders by a borrower.}\)
predict that delayed liquidation or “passivity” by multiple lenders would become more prevalent in countries where court-ruled liquidation procedures prevail; in particular, where creditors fear precipitating a liquidation procedure which expose them to the arbitrariness and slowness of the judicial system — somewhat recalling the arguments in Diamond (2004).

Finally, the tendency to refuse credit renewals — or the tendency to premature liquidation — is exacerbated during recessions and lessened during booms (Rajan, 1994; Thakor, 2005). According to our model, a reason why multiple lenders would go more often for liquidation during slumps may be that it is then more difficult to disentangle the idiosyncratic shocks to firms from the general shocks to the sector or the economy. This implies a lower precision of information available to the main bank or relationship bank in the loan. When the information is evenly distributed and imprecise (i.e, $\theta$ is small) the reliance mechanism tends to break down and the smaller or less informed lenders will precipitate a run. The identification of a non-performing loan as a lemon, by contrast, is quite easy during booms, through comparisons with similar business, so that the main bank’s information allows a precise sorting and liquidation is delegated.

Appendix

Proof of Proposition 1: From (1) and (5), when poorly informed, bank $i$ plays $res$ if

$$
(1 - \theta) [\alpha_j(y)L + (1 - \alpha_j(y)) \overline{\pi}(y)] + \theta E[\max(\pi(y, Z), L) | y] \geq L. \quad (15)
$$

If $\overline{\pi}(y) \geq L$, the condition is satisfied for all $\alpha_j(y)$. Consider next the case where $\overline{\pi}(y) < L$. When $\theta \geq \overline{\theta}(y)$ as defined in the proposition, the condition
(15) holds for \( \alpha_j(y) = 0 \). Moreover, the left-hand side of (15) is increasing in \( \alpha_j(y) \). Hence, the condition also holds for all \( \alpha_j(y) \), which means that \( res \) is a dominant strategy for bank \( i \). Finally, consider the case where \( \pi(y) < L \) and \( \theta < \tilde{\theta}(y) \). Condition (15) then does not hold if \( \alpha_j(y) = 0 \). The best response to the pure strategy \( res \) is therefore the pure strategy \( for \). Now, (15) obviously holds if \( \alpha_j(y) = 1 \); moreover, if \( \theta > 0 \), the condition holds as a strict inequality because \( E[\max(\pi(y, Z), L) \mid y] > L \) by Assumption 1. Thus, \( res \) is itself the best response to \( for \), thereby proving equilibrium \( P \). From the last argument, when \( 0 < \theta < \tilde{\theta}(y) \), there exists \( \alpha_j(y) \in (0, 1) \) such that (15) holds as an equality. Solving for \( \alpha_j(y) \) yields (7) and proves equilibrium \( M \). 

**Proof of Lemma 1:** By Assumption 1, \( E[\max(\overline{X}_b, L) \mid \pi_p] > L \). The denominator in (8) is therefore positive for all \( \pi_p \leq L \) and \( \tilde{\theta}(\pi_p) \) is well defined over the interval \([\pi_p^{min}, L]\), where \( \pi_p^{min} \) is the worst possible expectation when information is poor. Obviously, \( \theta_c = \tilde{\theta}(\pi_p^{min}) \) is positive and less than unity while \( \tilde{\theta}(L) \) is zero. Moreover, \( \tilde{\theta}(\pi_p) \) is strictly decreasing. Indeed, because \( \max(\overline{X}_b, L) \) is nondecreasing in \( \overline{X}_b \), Assumption 2 implies that \( E[\max(\overline{X}_b, L) \mid \pi_p] \) is nondecreasing in \( \pi_p \). Hence the sign of \( \partial \tilde{\theta}(\pi_p)/\partial \pi_p \) is the same as the sign of

\[
- \left( E[\max(\overline{X}_b, L) \mid \pi_p] - L + (L - \pi_p) \frac{\partial E[\max(\overline{X}_b, L) \mid \pi_p]}{\partial \pi_p} \right) < 0.
\]

**Proof of proposition 3:** The second-best strategies \( \alpha_1(\cdot) \) and \( \alpha_2(\cdot) \) maximize \( \tau(\theta) \) as defined in (10). It is easily seen that the solution to this problem is obtained by maximizing with respect to \( \alpha_1(\pi_p) \) and \( \alpha_2(\pi_p) \), for all \( \pi_p \), the expression:

\[
G(\alpha_1(\pi_p), \alpha_2(\pi_p), \pi_p) = \begin{cases} 
(1 - \theta)\theta \left\{ \alpha_1(\pi_p)L + (1 - \alpha_1(\pi_p))E[\max(\overline{X}_b, L) \mid \pi_p] \right\} \\
+ \theta(1 - \theta) \left\{ \alpha_2(\pi_p)L + (1 - \alpha_2(\pi_p))E[\max(\overline{X}_b, L) \mid \pi_p] \right\} \\
+ (1 - \theta)^2 \left[ L + (1 - \alpha_1(\pi_p))(1 - \alpha_2(\pi_p))\pi_p - L \right].
\end{cases}
\]
subject to \( \alpha_i(x_p) \in [0, 1], i = 1, 2 \). Let \( \mu_i(x_p) \) be the multiplier associated with the constraint \( \alpha_i(x_p) \leq 1 \) and \( \nu_i(x_p) \) the multiplier associated with \( \alpha_i(x_p) \geq 0 \). Omitting the arguments, the Lagrangian is

\[
L = G + \mu_1(1 - \alpha_1) + \nu_1\alpha_1 + \mu_2(1 - \alpha_2) + \nu_2\alpha_2.
\]

Writing \( H = E[\max(\overline{X}_b, L) | x_p] \) to simplify notation, the necessary conditions for a maximum are the Kuhn-Tucker first-order conditions

\[
\begin{align*}
\partial L/\partial \alpha_1 &= (1 - \theta)[(1 - \theta)(L - x_p)(1 - \alpha_2) - \theta(H - L)] - \mu_1 + \nu_1 = 0, \quad (16) \\
\partial L/\partial \alpha_2 &= (1 - \theta)[(1 - \theta)(L - x_p)(1 - \alpha_1) - \theta(H - L)] - \mu_2 + \nu_2 = 0, \quad (17)
\end{align*}
\]

together with complementary slackness and non-negativity of the multipliers,

\[
\mu_i(1 - \alpha_i) = \nu_i\alpha_i = 0, \quad \mu_i \geq 0, \quad \nu_i \geq 0, \quad i = 1, 2. \quad (18)
\]

Note that \( H > L \). Therefore, when \( x_p \geq L \) or when \( x_p < L \) and \( \theta > \widehat{\theta}(x_p) \) as defined in (8), \((1 - \theta)(L - x_p)(1 - \alpha_i) - \theta(H - L) < 0 \) for all \( \alpha_i \). There is then only one solution to (16), (17) and (18) and it involves \( \nu_i > 0 \), implying \( \alpha_i = 0, i = 1, 2 \). We henceforth discuss the case \( x_p < L \) and \( \theta < \widehat{\theta}(x_p) \).

We first discard the possibility of corner solutions of the form \( \alpha_1 = \alpha_2 = 0 \) or \( \alpha_1 = \alpha_2 = 1 \). Consider the first possibility. With \( \alpha_1 = \alpha_2 = 0 \), the term in brackets in (16) and (17) is positive since \((1 - \theta)(L - x_p) - \theta(H - L) > 0 \) for \( \theta < \widehat{\theta}(x_p) \). The conditions are therefore satisfied only if \( \mu_1 > 0 \) and \( \mu_2 > 0 \), which in turn implies \( \alpha_1 = \alpha_2 = 1 \), a contradiction. Similarly, noting that the term in brackets is negative if \( \alpha_1 = \alpha_2 = 1 \), the conditions are then satisfied only if \( \nu_1 > 0 \) and \( \nu_2 > 0 \), which implies \( \alpha_1 = \alpha_2 = 0 \), again a contradiction.

We now show that the conditions are satisfied by a corner solution of the form \( \alpha_1 = 1 \) and \( \alpha_2 = 0 \). By the above argument, \( \alpha_2 = 0 \) in (16) implies \( \mu_1 > 0 \) and therefore \( \alpha_1 = 1 \). In (17), \( \alpha_1 = 1 \) implies \( \nu_2 > 0 \), hence \( \alpha_2 = 0 \). This corner solution corresponds to the equilibrium \( P \).
Finally, it is easily seen that the term in brackets in (16) and (17) is zero if \( \alpha_1 = \alpha_2 = \alpha^* \), where the latter is as defined in (7). Together with \( \mu_i = \nu_i = 0, i = 1, 2 \), this therefore constitutes another possible solution to the set of necessary conditions. It is the only interior solution and corresponds to the equilibrium \( M \).

To conclude the proof for the case \( \pi_p < L \) and \( \theta < \tilde{\theta} (\pi_p) \), we therefore need to compare the strategies \( P \) and \( M \). Substituting in the definition of \( G \), the \( P \) strategies yield

\[
G_P(\theta) = (1 - \theta)(L + \theta H),
\]

where the resulting value of \( G \) has been written as a function of \( \theta \). Noting that the \( M \) strategies satisfy

\[
1 - \alpha^* = \frac{\theta H - L}{1 - \theta L - \pi_p}
\]

and substituting in the definition of \( G \) yields

\[
G_M(\theta) = 2\theta(1 - \theta) \left( L + \frac{\theta(H - L)^2}{(1 - \theta)(L - \pi_p)} \right) + (1 - \theta)(L - \theta H).
\]

Therefore

\[
\Delta(\theta) \equiv G_P(\theta) - G_M(\theta) = 2\theta(1 - \theta)(H - L) \left[ 1 - \frac{\theta(H - L)}{(1 - \theta)(L - \pi_p)} \right].
\]

This function is a quadratic in \( \theta \), with roots at \( \theta = 0 \) and \( \theta = \tilde{\theta}(\pi_p) = (L - \pi_p)/(H - \pi_p) \). For \( \theta \in (0, \tilde{\theta}(\pi_p)) \), \( \Delta(\theta) > 0 \) implying that the \( P \) strategies solve the maximization problem.

References


