Centralized or Decentralized Information: Which is Better for Providing Incentives?

Silvia Sonderegger*

Abstract

A risk-neutral ruler must invest in improving the quality of his country's infrastructures. Higher-quality infrastructures increase the profitability of capital investment by foreign entrepreneurs. The ruler wishes to maximize the amount of capital investment that flows into the country. Before selecting their investment, entrepreneurs receive a signal on the quality of infrastructures. We consider two cases. First, all entrepreneurs observe the same signal (Centralized Information). Second, each entrepreneur receives an independently drawn signal (Decentralized Information). We compare the effectiveness of these two scenarios for incentivizing the ruler. We find remarkably clear-cut results. When the entrepreneurs' investments are strategic complements, centralized information does a better job in incentivizing the ruler. The opposite holds when investments are strategic substitutes. This may help understand the role of media, rating agencies, public announcements and ambiguity. **JEL codes: D82, D62.**

*University of Bristol and CMPO. Address: Department of Economics, School of Economics, Finance and Management, 12 Priory Road, Bristol BS8 1TN (UK); e-mail: S.Sonderegger@bristol.ac.uk.

1 Introduction

Consider a country's business environment, or the quality of a country's infrastructures. Information about these characteristics may circulate in different ways. Data gathering may be relegated to an international watchdog, which issues publicly available country ratings. This is a case of centralized information. Or, entrepreneurs may hire their own experts, who gather and interpret data for them. This is a case of decentralized information. Which of these two scenarios is more effective for incentivizing a country's ruler to invest in ameliorating his infrastructures?

Suppose that the precision of information available to entrepreneurs is the same under both centralization and decentralization, and that the ruler is risk-neutral and aims at maximizing aggregate investment. If entrepreneurs take their investment decisions based only on their beliefs on the quality of infrastructure (i.e. independently of the actions of others) then centralization and decentralization are equivalent. For a given quality of infrastructures, they both deliver the same aggregate investment in expectation. The ruler's incentives to invest are therefore independent of the way in which information about the quality of the country's infrastructures is distributed.

Things are less clear if we allow for strategic interactions among the entrepreneurs. With strategic interactions, the entrepreneurs' investment decisions are affected not only by their beliefs about the quality of the country's infrastructure, but also by their beliefs about the actions of others. For instance, the entrepreneurs' actions (investments) may be strategic substitutes. Consider a country for which investment by foreign entrepreneurs takes primarily the form of outsourcing of low-skilled manufacturing activities. Investment by other entrepreneurs will drive wages up, making individual investment less profitable. In this case, if an entrepreneur believes that others will invest heavily in the country, he is less inclined to invest, as returns from his investment are smaller. Alternatively, strategic complementarities may exist between the entrepreneurs. This case may emerge if there are knowledge or technology spillovers between different enterprises. Consider a country for which investment by foreign entrepreneurs takes primarily the form of locating R&D and technology-intensive production facilities in that country. Although the wage-hiking effect described above is still present, entrepreneurs may now also benefit from investment by other entrepreneurs, through R&D and knowledge spillovers.[1] If this latter effect is sufficiently strong, entrepreneurs will be more inclined to invest in the country if they believe that others are also going to invest in it, as returns from investment are greater.

This paper investigates the effect of information structure (centralized/decentralized) on the ruler's incentives. We show that, with strategic interactions among the entrepreneurs, the ruler's incentives under centralization differ from his incentives under decentralization. Essentially, this is because information structure affects the *responsiveness* of aggregate investment to changes in the quality of the country's infrastructures. In turn, this generates different marginal benefits of investment for the ruler.

Key to the result is the effect of information structure on the *distribution of beliefs* across the population of entrepreneurs. Consider for instance the case in which information is centralized. Each entrepreneur knows that everyone else has received the same information as him (and everybody knows that everybody knows etc. that this is the case). Beliefs over the quality of infrastructure are homogeneous: all entrepreneurs hold the same beliefs. Now consider the case of decentralized information dissemination. Each entrepreneur receives a different piece of information (or individual signal). Although these individual signals are all drawn from the same distribution – they are all unbiased signals of the quality of the country's infrastructure – their realizations may differ. Beliefs are heterogeneous: different entrepreneurs may hold different beliefs (and everybody knows that everybody knows etc. that this is the case).

Suppose that there exist strategic complementarities among the entrepreneurs. Under centralization, if the common information available to all is favorable (i.e. it suggests that the quality of infrastructures is likely to be high) an entrepreneur has two reasons for investing, stemming from two different effects. First, the direct effect: keeping everything else equal, more favorable information suggests that investment is more likely to be profitable. So each entrepreneur is incen-

[1] See for instance Andretsch and Feldman (1996) for a discussion of how R&D spillovers may induce firms to cluster geographically.

tivized to invest. Second, the indirect effect: the entrepreneur knows that, through the direct effect, everyone else must have invested quite generously in the country. Because of strategic complementarities, this induces him to invest even more, and so on. This generates a multiplier effect. The final outcome is that aggregate investment following favorable information is considerably higher than following unfavorable information. There is a high responsiveness of aggregate investment to information – and therefore, indirectly, to the quality of infrastructures.

Now consider decentralized information. The direct effect of a given signal realization is the same as above. However, the indirect effect is now weaker. Even if an entrepreneur has received favorable information, he knows that this may not be true of everyone else. Some people may have received unfavorable information. So the entrepreneur's incentives to invest are smaller than in the centralized case. There is low responsiveness of investment to information. We conclude that, with strategic complementarity, the ruler's incentives to invest are greater under centralized, as opposed to decentralized, information. The opposite holds with substitutability. In that case, the ruler's incentives are greater under decentralization.

This paper therefore points to a very specific role of public announcements – and, generally, of all information that is released in a centralized manner, such as information coming from the media, rating agencies etc – : that of increasing the homogeneity of beliefs in the population (and ensuring that everybody knows that everybody knows etc. that this is so). The idea is that the release of information has two, distinct aspects. First, there is the *content* of the piece of information that an agent may receive. Second, there is information about *what others know.* This changes depending on whether the information is released in a centralized or decentralized manner. As noted by the literature on global games[2], when multiplier effects – arising from strategic interaction between agents – are present, even a small grain of doubt about the beliefs of others can have a large effect. With strategic complementarities, homogeneity of beliefs generates larger sensitivity of actions to signal realization, since individuals wish to coordinate with others. Hence, centralization is better for incentivizing the ruler. This finding may help understand why public events that put a country

[2] Such as Morris and Shin (2002), and Angeletos and Pavan (2004, 2007).

in the spotlight may be very powerful in providing incentives to the rulers of that country – think for instance of China and the 2008 Olympics.[3] Similarly, this result explains why authorities wishing to avoid public protests or demonstrations – activities where complementarities between participants are strong – may be willing invest more resources in those sectors (such as security) where failures or bad outcomes have a very public character. This implies that more effort and resources are spent on, say, tracking possible terrorist activities than on policies where failures are less public in nature, such as for instance pensions. This may be the case even when, on efficiency grounds, the same resources should be spent on each sector.

In contrast, under strategic substitutabilities, individuals do not wish to coordinate with others. Here, homogeneity of beliefs makes actions less responsive to signal realization. In this case, therefore, the release of information through a number of different sources is more effective for providing incentives than a single, centralized source of information.

Our conclusions lead us to various policy implications. Suppose that the entrepreneurs' actions are strategic complements. In this case, the most effective method for incentivizing the country's ruler is through the creation of a watchdog institution, which analyzes data over the quality of the country's infrastructures and makes public announcements of its conclusions. Now suppose instead that the entrepreneurs' actions are strategic substitutes. In this case, releasing data directly to the public is a more effective method of incentivizing the country's ruler. Since different investors may give different interpretations to the data available, this method of releasing information leads to greater uncertainty over the distribution of beliefs among the entrepreneurs, and makes the entrepreneurs' actions more responsive to their information.

Finally, it is clear that our results apply to a whole variety of contexts, such as organizations, communities, and essentially all environments that exhibit strategic interactions among its members. Some alternative applications are discussed in greater detail in Section 4.

[3] See for instance "Beijing Building the Olympic Dream", *BBC News,* 20 November 2006 (available at http://news.bbc.co.uk/2/hi/asia-pacific/6164330.stm) for a discussion of the vast programs of regeneration that have been prompted by the coming Olympics, and how this has been seen as an opportunity for the Communist Party to re-brand its public image.

Related Literature The framework we consider shares similarities with the models analyzed by Morris and Shin (2002) and Angeletos and Pavan (2004, 2007a) and b)). These papers also consider environments with strategic interactions among the players, and discuss the role played by information in shaping the players' "higher order beliefs" – namely, players' beliefs about other players' beliefs about other players' beliefs etc.– and their equilibrium actions. However, this literature concentrates on the welfare properties of information – what they term "*the social value of information*". In particular, their focus is on the effects of changes in the precision of public signals when agents also possess private information, and on the resulting trade-off between aggregate volatility – which increases in the public signal's precision – and cross-sectional dispersion – which decreases as the public signal becomes more precise. This paper builds on this literature but has a different focus, namely the incentive properties of centralized versus decentralized information transmission. The emphasis is on the incentive implications of different methods for delivering information.

Cornand and Heinemann (2007) add to the literature on the social value of information, by studying the optimal degree of publicity of the public signal. They show that it may be optimal to provide information with an intermediate degree of publicity – for instance, by exposing only a fraction of agents to public information. As made clear in Section 4, this differs from the prescriptions we derive. To maximize incentives, information should be either entirely centralized, or entirely decentralized.

Greif, Milgrom and Weingast (1994) concentrate on the role of centralized information transmission for incentives when the ruler is risk-averse. In their model, strategic interactions among the agents (entrepreneurs in the present context, merchants in theirs) are ruled out. The difference between centralized and decentralized information transmission for providing incentives rests on the different dispersion of *actions* that is generated in each case. Centralization generates less dispersion, and therefore imposes more risk on the ruler. When the ruler is risk-averse, centralized information transmission is therefore more effective for incentives. As we discuss below, the rationale for our results is different. What distinguishes one method of information transmission from the other is the different distribution of *beliefs* that they generate.

Our paper is also connected with Komai, Stegeman and Hermalin (2007), who show that, within a team, centralizing information in the hands of a single leader may induce workers to exert more effort. The rationale for their result is that, in the centralized scenario, the information available to workers is coarser than under dissemination – since leaders may only credibly convey their information through their choice of a binary action.

Finally, the common knowledge role of public or centralized announcements is informally discussed in Chwe (1998, 1999).

The remainder of the paper is organized as follows. In Section 2, we introduce the model. As this allows us to make the intuitions for the results clearer, we initially assume that the entrepreneurs' payoff functions take a specific form (the same as in Angeletos and Pavan 2004), and that information may either be entirely centralized, or entirely decentralized. In Section 3 we derive and discuss our main results. In Section 4, we relax some of the restrictions imposed earlier. In particular, we allow for intermediate levels of centralization, and also derive the sufficient conditions for our results to go through with more general entrepreneur payoff functions. We also discuss some alternative applications of our results, beyond the specific application considered in the main body. Section 5 concludes. All the proofs can be found in the Appendix.

2 Model

Payoffs The players of the game we consider are a country's ruler and a continuum of measure one of foreign entrepreneurs, indexed by i and uniformly distributed over the $[0,1]$ interval. We utilize the same payoff functions as Angeletos and Pavan (2004).[4] Namely, entrepreneurs have utility

$$u_i = Ak_i - 0.5k_i^2 \tag{1}$$

[4] As mentioned in the introduction, in Section 4 we discuss the robustness of our results to more general entrepreneurs' utility functions.

where $k_i \in \mathbb{R}$ is individual i's level of investment, A is the return to investment, $0.5k_i^2$ is the cost to investment. Let $K = \int_0^1 k_i di$ denote aggregate level of investment. The return of investment for each single entrepreneur is

$$A = \theta K + \omega \tag{2}$$

where θ parametrizes the nature and magnitude of strategic interactions among entrepreneurs, and $\omega \in \{h, l\}$, $h > l \geq 0$, indicates the (unobservable) underlying fundamental of the country's economy (or state of the world). We assume that $\theta \neq 0$ and $\theta < 1$. The restriction $\theta \neq 0$ ensures that there are strategic interactions among the entrepreneurs. $\theta > 0$ applies to situations where there are investment complementarities. These may for instance arise if there are technological spillovers between the economic projects in which the entrepreneurs invest their capital. $\theta < 0$ applies to situations where there are substitutabilities. These may arise if the projects in which the entrepreneurs invest are competing with one another, or generally if there are negative externalities between them. The restriction $\theta < 1$ ensures that complementarities are not so strong that there are multiple equilibria; therefore, the equilibrium that we characterize is the unique equilibrium.

Let $a \in \mathbb{R}^+$ represent the amount of resources invested by the ruler in the country's infrastructures. For a given amount of resources a, the underlying fundamental of the economy is

$$\omega = \begin{cases} h \text{ with probability } p(a) \\ l \text{ with probability } 1 - \ p(a) \end{cases} \tag{3}$$

where $p(.)$ is a strictly increasing, concave function, with $p(0) > 0$.[5] The net cost to the ruler of investing a is $C(a)$, where $C(.)$ is a strictly increasing convex function, with $C(0) = C'(0) = 0$. The assumption that $C(a)$ is increasing implies that, for the ruler, the direct costs of higher quality infrastructures outweigh any direct benefit.

Aggregate investment matters to the ruler because it generates positive externalities, and in-

[5] The assumption that $p(0) > 0$ allows us to rule out "bad" equilibria where $a = 0$.

creases the country's welfare. The ruler maximizes

$$E\left(K \mid a\right) - C(a)$$

where $E\left(K \mid a\right)$ indicates the expected aggregate investment conditional on a.

Information Entrepreneurs do not observe the ruler's choice of a. However, they have access to imperfect information about the realization of the state of the world. We consider two cases. In the first – the *Centralized Information* case – all individuals observe the same signal s. The signal's realization depends on the state of the world, as follows:

$$\begin{aligned} &\text{when } \omega = h : s = \begin{cases} L \text{ with probability } \lambda \\ H \text{ with probability } 1-\lambda \end{cases} \\ &\text{when } \omega = l : s = \begin{cases} H \text{ with probability } \lambda \\ L \text{ with probability } 1-\lambda \end{cases} \end{aligned} \tag{4}$$

for an exogenously given $\lambda \in (0, 0.5)$.

In the second case – the *Decentralized Information* case – *each* individual observes a private signal s_i. Signals are drawn independently, but their distribution depends on the realization of ω. Namely:

$$\begin{aligned} &\text{when } \omega = h : s_i = \begin{cases} L \text{ with probability } \lambda \\ H \text{ with probability } 1-\lambda \end{cases} \\ &\text{when } \omega = l : s_i = \begin{cases} H \text{ with probability } \lambda \\ L \text{ with probability } 1-\lambda \end{cases} \end{aligned} \tag{5}$$

where s_i is drawn independently of s_j, for $i \neq j$, and λ is the same as in (4). This ensures that the precision of the information available to each entrepreneur is the same under both centralization and decentralization. In our analysis, we can therefore concentrate on the specific effects of different information structures (centralized/decentralized), as opposed to the effects of different information technologies.

Timing The timing of the game is as follows:

t=0: The ruler selects an amount $a \in \mathbb{R}^+$ of resources to invest in the country's infrastructures.

t=1: Given a, Nature selects ω according to (3).

t=2: Under centralized information: all entrepreneurs observe a common signal s, drawn according to (4). Under decentralized information: each entrepreneur receives a private signal s_i, drawn according to (5).

t=3: All entrepreneurs simultaneously select a level of investment $k_i \in \mathbb{R}$.

t=4: Payoffs are realized.

The game we consider is therefore of a structure similar to the games analyzed by Morris-Shin (2002) and Angeletos-Pavan (2004, 2007 (a) and (b)), with an added initial stage ($t = 0$). In this initial stage the country's ruler selects a, which determines the distribution from which the state of the world is drawn.

Finally, it should be stressed that we only consider a one-shot game, and rule out the possibility of incentives being provided through repeated interaction between the players.[6]

3 Results

We solve the game by backward induction. First, we characterize the entrepreneurs' optimal strategy in both the centralized and decentralized cases, for a given a selected by the ruler. We then analyze the ruler's optimal choice of a in each case.

[6] Notice however that because information is imperfect, in our model punishment would occur along the equilibrium path in any trigger-strategy equilibrium of the repeated game. The use of trigger strategies may thus be a very expensive way of disciplining the ruler. Moreover, punishing the ruler by withdrawing their investment would hurt the entrepreneurs. Hence, punishment would here be vulnerable to renegotiation. This may considerably limit the set of outcomes that can be sustained as equilibria of the repeated game (see for instance Farrell and Maskin 1989).

The optimal choice of k_i for an individual who has observed a signal with realization $\varsigma = H, L$ is

$$k(\varsigma) = E(\omega \mid \varsigma) + \theta E(K \mid \varsigma) \tag{6}$$

where $E(\omega \mid \varsigma)$ indicates an entrepreneur's expectation over the value of the fundamental ω when he has observed a signal with realization $\varsigma = H, L$, and $E(K \mid \varsigma)$ indicates his expectation over aggregate investment K. From (6), it is straightforward to see that if each entrepreneur's payoff were independent of aggregate investment – i.e. if $\theta = 0$ – then at equilibrium each entrepreneur would simply select a level of investment equal to his expectation over ω, given the information at his disposal. In both the centralized and decentralized information cases, the entrepreneurs' follow the same updating rule. For a given equilibrium a selected by the ruler, this is given by

$$E(\omega \mid H) = \frac{p(a)(1-\lambda)h + (1-p(a))\lambda l}{\lambda + p(a)(1-2\lambda)} \tag{7}$$

$$E(\omega \mid L) = \frac{(1-p(a))(1-\lambda)l + p(a)\lambda h}{1-\lambda-p(a)(1-2\lambda)} \tag{8}$$

Hence, with $\theta = 0$ the *responsiveness* of individual investment to signal realization would be the same in both the centralized and decentralized case. As the ruler is risk-neutral with respect to aggregate investment, he would see no difference between the two cases. His incentives to invest would be the same, independently of whether the entrepreneurs' information is centralized or decentralized.

This equivalence between centralized and decentralized information ceases to hold once we allow for strategic interactions between the entrepreneurs – i.e $\theta \neq 0$. Although the entrepreneurs' updating rules are the same in both cases, their equilibrium strategies differ. This in turn generates different incentives for the ruler. Lemma 1 characterizes the entrepreneurs' equilibrium strategies, in the two cases of centralized and decentralized information.

Lemma 1 (Entrepreneurs' Equilibrium Behavior): *For a given a, the unique Nash equilibrium has:*

$$k(H)-k(L)=\begin{cases}\frac{E(\omega|H)-E(\omega|L)}{1-\theta}\ \textit{in the centralized case}\\ \alpha(a)\left[E(\omega\mid H)-E(\omega\mid L)\right]\ \textit{in the decentralized case}\end{cases}$$

where $\alpha(a)\equiv\frac{[p(a)(2\lambda-1)+1-\lambda][\lambda+p(a)(1-2\lambda)]}{p(a)(2\lambda-1)^2(1-\theta)(1-p(a))+\lambda(1-\lambda)}$.

Lemma 1 illustrates how the *responsiveness* of investment to signal realization differs in the centralized and decentralized information cases. First, consider centralization. The difference between the amount invested by each entrepreneur when the common signal realization is H and the amount invested when it is L is

$$\frac{E(\omega\mid H)-E(\omega\mid L)}{1-\theta} \tag{9}$$

Now consider the decentralized case. The difference between the amount invested by an entrepreneur when the realization is of his private signal is H and the amount he invests when it is L is

$$\alpha(a)\left(E(\omega\mid H)-E(\omega\mid L)\right) \tag{10}$$

It is straightforward to verify that $\alpha(a)<1/(1-\theta)$ for $\theta>0$, and vice-versa. With strategic complementarity – namely, $\theta>0$ – the responsiveness of individual investment to signal realization is greater in the centralized case. The opposite holds with strategic substitutability – i.e. $\theta<0$. In that case, the effect of signal realization on investment choice is greater under decentralization.

To see the rationale for the result, take for instance a situation where there are strategic complementarities between the entrepreneurs (a similar rationale holds with strategic substitutability). Consider an individual i who has observed a signal with high realization. The difference between the centralized and decentralized information arises because while in the former case i knows that the *entirety* of entrepreneurs has received his same signal, in the latter case he knows[7] that a

[7] The fact that the individual *knows* that a positive share of the population must have received a signal different to his is an artefact of our assumption of a continuum of agents. With a discrete number of agents – say, for instance, two – each individual would simply know that *with a positive probability* the other person's information is different from his. This "grain of doubt" would however be sufficient to generate the result.

positive share of the population has received a low signal, and will therefore hold beliefs over ω characterized by $E(\omega \mid L)$. Given the existence of complementarities, this induces i to place a positive weight on $E(\omega \mid L)$ when selecting his investment. $E(\omega \mid H)$ therefore has a smaller weight in determining $k(H)$ than in the centralized case.

It is interesting to note that, in addition to the initial impulse, the effect described above is further strengthened as i acknowledges that all the other individuals who have received a high signal will also place a positive weight on $E(\omega \mid L)$ when selecting their investment. So i should accordingly increase the weight he assigns to it a little further, and so on. The end result of this process is that $E(\omega \mid L)$ may ultimately have a rather large weight, even if signals are very precise.

The differences between the centralized and decentralized setting highlighted in Lemma 1 translate into different incentives for the ruler. Intuitively, the ruler is more motivated to invest resources in the economy when aggregate investment is more responsive to the realization of the fundamental. The information structure (centralized/decentralized) that maximizes the ruler's incentives is therefore the one, which generates greater investment responsiveness to signal realization. Proposition 1 makes this point precise.

Proposition 1 (Ruler's Incentives): *The Bayesian Nash equilibrium level of investment by the ruler is uniquely defined under both centralization and decentralization. With strategic complementarity, the amount of resources invested by the ruler at equilibrium is strictly greater when the entrepreneurs' information is centralized rather than decentralized. The opposite holds with strategic substitutability.*

The presence of strategic interactions between the entrepreneurs generates different degrees of responsiveness of investment to signal realization. In turn, this affects the ruler's incentives. Different structures of entrepreneurs' information therefore generate different levels of investment in the country's infrastructure by the ruler. This may affect the welfare comparisons between centralized and decentralized environments.

To see this, consider for instance the following numerical example[8]: $h = 0.75$, $l = 0.25$, $\theta = 0.5$ and $\lambda = 0.1$. First, suppose that the probability that $\omega = h$ (and the corresponding probability that $\omega = l$) is exogenously given, and equal to p (respectively, $1 - p$). In this case, aggregate welfare is always maximized under decentralization. Things change if we allow for p to be determined endogenously, as a function of the ruler's investment in the country's economy. As explained in Proposition 1, with strategic complementarity centralization is more effective for incentivizing the ruler. Suppose that $p(a) = 0.5a + 0.1$, and $C(a) = 0.5a^2$. It is straightforward to show that aggregate welfare is always greater under centralization. Although other elements – such as the dispersion of the entrepreneurs' actions – still play a role, the effect of centralization on the ruler's incentives is sufficiently strong to alter the trade-off between centralization and decentralization, making the former unambiguously better.

The results highlighted in Proposition 1 are rather intuitive. When complementarities exist among the entrepreneurs – i.e. entrepreneurs wish to coordinate with one another – centralization is more effective at disciplining the ruler. Essentially, this is because centralization *coordinates the entrepreneurs' beliefs* over the state of the world. In turn, this results in investment being more sensitive to signal realization. Vice-versa, with substitutabilities – i.e. when entrepreneurs do not wish to coordinate too much with one another – decentralized information is more effective for incentives. This is because decentralization creates *heterogeneity in the entrepreneurs' beliefs* about the state of the world.

It is important to stress that the key element here is the effect of information structure on the homogeneity/heterogeneity of entrepreneurs' beliefs. This is different from saying that information structure matters because it affects the dispersion of individual investment levels. To see this point, consider a population of myopic entrepreneurs, who erroneously believe that everyone else has observed the same signal as theirs, even in the decentralized case. It is straightforward to show that in this scenario the entrepreneurs' equilibrium strategies would be the *same*, independently of information structure – just as in the case, discussed above, where $\theta = 0$. Having observed a

[8] Details are omitted in the interest of space, but are available from the author upon request.

signal with realization $\varsigma = H, L$, an entrepreneur would select $k(\varsigma) = \frac{E(\omega|\varsigma)}{1-\theta}$. For a given a, expected aggregate investment would then be the same under both centralization and decentralization. Since the ruler is risk-neutral, his incentives would then be equal in both cases. This would occur in spite of the fact that the amount of dispersion in individual investment levels differs from one case to the other. What matters is therefore the homogeneity of *beliefs*, not *actions*. With strategic complementarity, centralization is more effective than decentralization in disciplining the ruler because it coordinates beliefs, not because it coordinates actions. Similarly, with strategic substitutability, decentralization performs better because it decreases coordination in beliefs, not because it decreases coordination in actions.

4 Extensions

4.1 Ambiguity

The insights derived in the previous sections can be applied to the analysis of language and communication. In particular, our model can help us gain a better understanding of the role of ambiguity and shared understanding of words and expressions. The key idea is that ambiguity may decrease the amount of common knowledge, and therefore affect the extent to which individuals react to their information. The importance of common knowledge and shared understanding in communication has been discussed by Morris and Shin (2006), who present a theory of optimal communication, based on the trade-off between precision of information on one hand, and the shared nature of that information on the other. Here we take a different approach, focussing on ambiguity as a mean to decrease common knowledge. As will become clear below, the trade-off we explore is therefore rather different from that in Morris and Shin (2006).

Consider a central agency, who possesses information about the state of the world. When passing this information to entrepreneurs, the agency can utilize one of two approaches. If can either deliver its information in an unambiguous manner, for instance by using scores, or it can deliver its information in a more ambiguous way. To see the difference between these two approaches, consider the problem faced by a teacher when writing references for one of his students. One way he may go about the task is to "harden" the information, for instance by quantifying the impression he has of the student. This approach is likely to deliver a reasonably unambiguous message. For instance, a statement such as "this student is in the bottom third of the distribution of those that I have taught so far" is bound to be understood by everyone as meaning that the student is pretty bad. Alternatively, the referee can use a "softer" tone. In this case, ambiguities are more likely to emerge over the precise content of the letter. A statement such as "person X is a reasonably able student" may be interpreted differently by different people. Although (at least in the UK) the majority of those reading this reference will understand that the student is, in fact, pretty weak, a few may misinterpret it, and read it in a positive light.

What are the effects of the agency using more or less ambiguous language on the incentives provided to the ruler? More generally, what are the trade-offs involved? These are the questions we address in this section. In order to do this, we consider a modification of the benchmark model of Section 2.

Consider an agency, who possesses (imperfect) information about the state of the world. More precisely, suppose that the agency observes a signal s, as in (4). The agency can communicate its information to entrepreneurs in two ways:

(i) First, the agency may use unambiguous language.

Using unambiguous language is equivalent to having all entrepreneurs observe s (and knowing that everyone else has observed s).

(ii) Second, the agency may use a more ambiguous language.

When some ambiguity is present, investors must decode the agency's message. We model this process of decoding as follows: any given entrepreneur understands the agency's message correctly with probability $1-\phi$, and understands it incorrectly with probability $\phi < 0.5$. That is, suppose that the agency has received a signal $s = H, L,$ and denote $\{H, L\} \setminus s$ as s'. Each entrepreneur i

observes a signal η_i given by

$$\eta_i = \begin{cases} s \text{ with probability } 1-\phi \\ s' \text{ with probability } \phi \end{cases}$$

This implies that if the agency has received a signal $s = H$ and sends a message to this effect using ambiguous language, a share $1-\phi$ of all entrepreneurs will correctly interpret the agency's message, i.e. will infer that $s = H$, while a share ϕ will interpret the message incorrectly, i.e. will infer that $s = L$. The value of ϕ parametrizes the severity of ambiguity, with a higher ϕ corresponding to greater ambiguity.

Case (i) is equivalent to the case of centralized information seen above. Case (ii) is similar to the case of decentralized information. Notice however that while in previous sections the precision of information was the same both under centralization and decentralization, here this is no longer the case. Introducing ambiguity endogenously adds noise, which results in information being transmitted less precisely. To see this, consider an entrepreneur, who has interpreted the information delivered by the agency as indicating that $s = H$. In case (i), the entrepreneur's posterior belief of the state of the world being h is

$$\Pr(\omega = h \mid \eta_i = H)^{no\ ambiguity} = \frac{p(1-\lambda)}{\lambda + p(1-2\lambda)} \tag{11}$$

while in case (ii) it is

$$\Pr(\omega = h \mid \eta_i = H)^{ambiguity} = \frac{p(1-\lambda-\phi(1-2\lambda))}{\lambda(1-\phi)+\phi(1-\lambda)+p(1-2\lambda)(1-2\phi)} \tag{12}$$

It is straightforward to verify that (11) > (12). In this case, therefore, ambiguity leads to a loss of precision.

What are the implications of ambiguity of the ruler's incentives? Applying the results of Proposition 1, we know that, keeping precision constant, introducing some ambiguity in the agency's message will increase the ruler's incentives whenever there are strategic substitutabilities among the entrepreneurs. In contrast to previous sections, however, we now have a trade-off, in that ambiguity also introduces additional noise. Keeping everything else equal, less precision makes the entrepreneurs more reluctant to act on the information at their disposal, and will therefore weaken the ruler's incentives. To sum up, ambiguity has two opposing effects. On one hand, it decreases the amount of common knowledge among the investors. When strategic substitutabilities are present, this is good, as it makes entrepreneurs more willing to react to their information. However, ambiguity also decreases the precision of the information transmitted to entrepreneurs. This makes the entrepreneurs react less to the their information, caeteris paribus.

Note that the trade-off we obtain is different from that discussed in Morris and Shin (2006). In M-S, greater precision may only be achieved by utilizing terms that are likely to be misunderstood by some individuals. For instance, the term "cardiac infarction" is more precise than "heart attack". However, the latter term is more easily understood by the majority of people. Here, we concentrate on a different type of trade-off. Shared understanding and precision go hand in hand. Saying "this country is doing reasonably well" is both less precise and generates less common understanding than saying "on a scale from 0 to 10, I rate this country at 6".

In terms of providing incentives, the outcome of the trade-off we analyze may go either way, depending on the strength of the two different effects. Consider for instance $p(a) = a + 0.1$, $C(a) = 0.5a^2$, $\theta = -1$, $\lambda = 0.25$, $\phi = 0.25$, $h = 0.75$ and $l = 0.25$. In this case, it is straightforward to verify that ambiguity increases the ruler's incentives, generating higher effort. However, things change if we have $\phi = 0.4$. In this case, the drop in precision arising from ambiguity is too severe, and clarity is better at incentivizing the ruler than ambiguity.

Notice that ambiguity may only be optimal for incentivizing the ruler in the presence of strategic substitutabilities. With strategic complementarities, clarity is always better than ambiguity, since it both facilitates coordination and increases precision.

4.2 Intermediate Degrees of Centralization

In previous sections, we have analyzed the two polar cases where information available to entrepreneurs is either *completely centralized* (i.e. a single watchdog institution releases a signal, observed by all) or *completely decentralized* (i.e. each investor hires his own consultant to gather data and interpret them). In practice, however, we may think of situation that are somewhere in between these two extremes. For instance, rather than a single watchdog institution we may have two, or three. Could these possibilities be more effective for incentives than either of the two cases seen above? We now explore this question by allowing for intermediate degrees of centralization.

Consider the following setting: entrepreneurs are divided into n groups. All those belonging to same group observe the *same* signal. Signals observed by different groups are drawn *independently*, as in (5). The scenarios analyzed in the previous sections – full centralization and full decentralization – are special cases of this more general specification: the first corresponds to $n = 1$, while the second corresponds to $n \to \infty$. The question we are asking is whether intermediate degrees of centralization can be more effective for incentivizing the ruler than either of these polar cases. Is there an interior "optimal" degree of centralization? Or are the special cases considered above the most effective for providing incentives? Proposition 2 shows that this latter hypothesis is indeed the correct one.

Proposition 2 (Intermediate Levels of Centralization): *Intermediate levels of centralization are never optimal for incentivizing the ruler.*

When complementarities exist among the investors, the ruler's incentives are maximized by setting $n = 1$. As n increases, incentives get weaker, and reach their weakest when $n \to \infty$. Indeed, the difference between the entrepreneurs' behavior when $n = 1$, and that when $n = 2$ may be rather large. So in this case the incentives provided by having, say, one single rating agency can be considerably greater[9] than those with two rating agencies, each providing information to half the entrepreneurs' population. More generally, this suggests that having two (possibly competing)

[9] As shown in the Appendix, for $\theta \to 1$ the difference difference between the two becomes infinitely large.

sources of public information may decrease the effectiveness of information disclosure as a mean to maintain discipline. Vice-versa, with substitutabilities, the ruler's incentives are maximized when $n \to \infty$. As n decreases, incentives get weaker, and reach their weakest when $n = 1$. Here, two rating agencies are always better than one.

Our conclusion is therefore rather sharp: individuals should either observe identical signals, or signals that are fully independently drawn. This may have implications in terms of organizational design. Consider a firm, composed of different divisions. Depending on the relationship between the tasks assigned to each division, performances may either be complements, or substitutes. The former case is likely to emerge when different divisions perform complementary tasks (e.g., one division is devoted to gathering data on consumer preferences, and the other is devoted to tailoring the company's product to meet those preferences), while the latter is likely to emerge when different tasks are substitutes (e.g., one division deals with sales and promotional effort, while the other deals with product quality). Division performance is also affected by the quality of the strategic decisions taken at CEO and senior management level (better decisions enhance the productivity of the effort and resources spent in each division). How should information about the quality of strategic decisions taken at top level be distributed between divisions? One possibility is that all heads of divisions be briefed in a centralized manner – for instance, in a meeting where everyone is present. Another possibility is to have information flow through one-to-one communication. In between these two alternatives we have hybrid combinations, such as meetings that include only a few heads of divisions at a time. Our results suggest that, to improve discipline at senior level, this latter alternative is never optimal. Information should either be entirely centralized (when divisions perform complementary tasks), or entirely decentralized (when divisions perform tasks that are substitutes).

Another application of our results concerns the role of the media. It is clear that the media do not act simply as a channel for delivering information, but also have a powerful function in shaping individual perceptions of what others know and think. Our results may shed light on the relationship between the media and government accountability. In the presence of complementarities, media attention can be a useful disciplining device – think for instance of the role played by media in prompting public demonstrations. Conditional of media independence being preserved, Proposition 2 suggests that in this case discipline is maximal with only one centralized source of information. Of course, monopolization of the media market may also present drawbacks, such as an increased likelihood of capture by the government (see for instance Besley and Prat 2006). What we suggest here, however, is that excessive fragmentation of the media market may also be problematic, by decreasing the amount of common knowledge and therefore making it harder for people to coordinate in response to bad policies. When strategic complementarities are present, this may soften government discipline.[10]

4.3 Robustness of Results

In Section 3, we have concentrated our attention on a very specific application. This has the advantage of making the intuitions behind our results easier to follow, but may raise questions over their generality. We now show that our results hold for a wider set of models than the specific application considered above.

Suppose that, instead of having utility (1), we let entrepreneur utility be more generally defined as $U(K, \omega, k_i)$, strictly concave in k_i. Restrict attention to utility functions that satisfy the following assumptions:

A1 Each entrepreneur's first order condition can be written as

$$f(E(K), E(\omega)) - k_i = 0 \tag{13}$$

where $f(.) \in \mathbb{R}^+$ is strictly monotone in $E(K)$ and strictly increasing in $E(\omega)$.

[10] Note however that reducing common knowledge may also present advantages, as pointed out by the literature on speculative attacks. For instance, Cornand and Heinemann (2007) show that, by making it harder for agents to coordinate, less common knowledge may help reduce the likelihood of self-fulfilling crises.

Assumption A1 may be satisfied only when K and ω enter independently in the entrepreneur utility function.[11] Essentially, this rules out cases where K and ω interact with each other (i.e., a higher ω enhances the effect of a large K on the productivity of individual investment or vice-versa). This restriction is standard in the literature on the value of information (see for instance Angeletos and Pavan (2007)).

A2 In both the centralized and decentralized case, for any given level of $p(a)$ there exist a unique equilibrium of the global game played by the entrepreneurs.

A3 The function $f(.)$ satisfies

$$\frac{df(E(K \mid H), E(\omega \mid H))}{dp(a)} - \frac{df(E(K \mid L), E(\omega \mid L))}{dp(a)} \text{ non-increasing in } p(a) \tag{14}$$

Proposition 3 (Robustness of Results): *Assumptions A1 to A3 are sufficient to ensure that Proposition 1 holds for more general entrepreneur utility functions.*

Proposition 3 shows that our results are actually quite general. Assumptions A2 and A3 guarantee a unique equilibrium of the overall game. Assumption A1 ensures that responsiveness to signal realization changes according to the degree of homogeneity of entrepreneurs' beliefs over ω. As seen in Section 3, this is the key element for obtaining our results.

4.4 Alternative Applications

Our insights apply to a wide range of situations, beyond the particular working example we utilize. Here we illustrate in some detail a couple of alternative applications.

[11] Examples include (1), $u_i = k_i \theta K - \frac{0.5k_i^2}{\omega}$ (discussed below), $u_i = k_i \left(K^\theta + \frac{1}{\omega}\right) - 0.5k_i^2$ and so on.

Blood Donors The National Health System (NHS) must invest resources to improve hospital facilities for donating blood. Greater investment decreases the probability that donors will experience inconvenience when donating their blood. The quality ω of blood donation facilities depends on a, the investment by the NHS, as in (3). Blood donors must decide how much blood to donate. Each donor commits to a certain supply, and cannot renege on this commitment. Higher-quality facilities decrease the personal cost of donation. Each donor derives utility from donating blood, which is increasing in the social value of his donation. The marginal social benefit of additional blood supply is decreasing in the aggregate quantity of blood being donated. Hence, there exist strategic substitutabilities between the donors. Before deciding how much blood to donate, each individual receives a signal on the quality of hospital facilities. Each individual i selects his blood supply k_i to maximize

$$(1+\theta E(K))k_i - \frac{0.5k_i^2}{E(\omega)} \tag{15}$$

where K indicates aggregate supply of blood, $1+\theta E(K)$ is the expected marginal social benefit of additional blood supply, and $\theta < 0$. Suppose that an individual has observed a signal with realization $\varsigma = H, L$. His optimal blood supply is

$$k(\varsigma) = E(\omega \mid \varsigma)(1-\gamma E(K \mid \varsigma)) \tag{16}$$

Notice that, in contrast to the application analyzed in the main body, here the individual's optimal choice of k_i is not additively separable in $E(\omega \mid \varsigma)$ and $E(K \mid \varsigma)$.

The NHS wishes to maximize the total quantity of blood supplies, minus the cost of investment. Its objective function is therefore equal to $E(K \mid a) - C(a)$.

It is straightforward to show that in this example there exist a unique equilibrium of the global game played by the donors. Moreover, at equilibrium, the sensitivity of individual blood supply to signal realization is greater under decentralized rather than centralized information. Hence, in this case, a centralized structure – where information over the quality of facilities is released through, for instance, a country-wide blood donors' association's website or bulletin– is less effective for incentivizing the NHS than a decentralized structure – where, for instance, information is made available but individual donors must gather it on their own, through hospital websites and the like. This may be the case even if the information available to individual donors is less precise than the information available to the donors' association, provided that the discrepancy in precision is not too large.

Criminal Activity The government of a country must invest in ameliorating the quality of his police force. Greater investment increases the probability of catching criminals. The quality ω of the police force depends on a, the investment by the government, as in (3). Individuals must decide on their supply of criminal activity (such as, for instance, tax evasion, or corruption). A high-quality police force decreases the marginal benefit of such activity. For a given quality of the police force, the returns to breaking the law are increasing in K, the aggregate amount of criminal activity. This strategic complementarity emerges because the authorities face a constraint on the number of convictions that can be made (arising, from instance, from a limited supply of prison places).[12] Before making his decision, each individual observes a signal on the quality of the police force, which determines the probability of catching someone breaking the law. Each individual i selects his supply k_i of criminal activity to maximize

$$(\theta E(K) - \omega)k_i - 0.5k_i^2 \tag{17}$$

where K indicates aggregate criminal activity, and $\theta > 0$. The government wishes to minimize K, plus cost of investment. Following our earlier analysis, here the most effective way to incentivise the government is through a centralized information system. Data about the quality of the police force should be released in the most centralized way possible. An independent Commission should

[12] Alternatively, strategic complementarity may emerge because an increase in overall rent-seeking or criminal activity makes rent-seeking more attractive relative to productive activity. See Murphy, Schleifer and Vishny (1993) for an account of how rent-seeking activities (interpreted broadly as activities that make property insecure) exhibit increasing returns. Blume (2004) points to another reason for strategic complementarities in criminal activities, stemming from social norms. As more people commit crimes, more people become tagged and the expected stigma cost of committing a crime falls, making crime more attractive.

be created, which monitors the quality of the police force, and releases ratings summarizing its findings.

5 Concluding Remarks

What are the differences between centralized and decentralized information disclosure? Should raw data be made available to all – so that each individual may interpret them as he wishes – or should data only be observed by a central body, who then publicly releases its own reading of them? This paper hopes to provide some insights to these questions. We concentrate on the incentives effects of centralization versus decentralization of information disclosure. Consider a ruler who must invest in improving his country's infrastructures, in order to attract foreign entrepreneurs. Are his incentives to invest greater when the information available to entrepreneurs is centralized or when it is decentralized? Now consider a government, who must invest in ameliorating the quality of its police force, to minimize criminal activity. Is a commitment by the government to release data about the quality of the police force a good idea? Would a Commission especially designed to monitor police quality – and who releases reports about its findings – be a more effective commitment device for the government? Are there any advantages to having a small number of public sources of information – such as a few television channels or a few newspapers? The answers to these questions depend on the effect that different information structures may have on the responsiveness of agents' actions to the information being released. We have characterized this effect, and showed that it differs, according to the nature of the strategic interactions between the agents. With strategic complementarity, a centralized information system generates greater responsiveness of actions to information, and therefore provides greater incentives. The opposite holds with strategic substitutability.

Overall, we believe that our insights will contribute to build a better understanding of the role of information – and the manner in which information is released – in environments with incomplete information.

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Appendix

Proof of Lemma 1

Centralized information

For a given realization s of the centralized signal, each entrepreneur selects

$$k(s) = \theta E(K \mid s) + E(\omega \mid s). \tag{18}$$

Note that, since we have a continuum of entrepreneurs, the equilibrium is necessarily symmetric. At equilibrium,

$$k = K = \frac{E(\omega \mid s)}{1-\theta}. \tag{19}$$

Hence,

$$k(H) - k(L) = \frac{E(\omega \mid H) - E(\omega \mid L)}{1-\theta}. \tag{20}$$

Decentralized information

Suppose that the realization of a individual i's private signal is $s_i = H$. The first order condition for utility maximization gives:

$$k(H) = \theta E(K \mid H) + E(\omega \mid H) \tag{21}$$

where

$$E(K \mid H) = \begin{array}{c} \Pr(\omega = h \mid H)\left[(1-\lambda)k(H) + \lambda k(L)\right] \\ + \Pr(\omega = l \mid H)\left[(1-\lambda)k(L) + \lambda k(H)\right] \end{array}. \tag{22}$$

Letting $\Pr(\omega = h \mid H) = p_H$, this can be rewritten as

$$E(K \mid H) = k(H)\left[p_H(1-2\lambda) + \lambda\right] + k(L)\left[(1-\lambda) - p_H(1-2\lambda)\right] \tag{23}$$

where $p_H \equiv \frac{(1-\lambda)p(\widehat{a})}{(1-\lambda)p(\widehat{a})+\lambda(1-p(\widehat{a}))}$ ($\widehat{a}$ indicates the entrepreneurs' conjecture about a; of course, at equilibrium this conjecture must be correct). Similarly, suppose that the realization of a individual i's private signal is $s_i = L$. The first order condition for utility maximization gives:

$$k(L) = \theta E(K \mid L) + E(\omega \mid L) \tag{24}$$

where

$$E(K \mid L) = \begin{array}{c} \Pr(\omega = h \mid L)\left[(1-\lambda)k(H) + \lambda k(L)\right] \\ +\Pr(\omega = l \mid L)\left[(1-\lambda)k(L) + \lambda k(H)\right] \end{array}. \tag{25}$$

Letting $\Pr(\omega = h \mid L) = p_L$, this can be rewritten as

$$E(K \mid L) = k(H)\left[p_L(1-2\lambda) + \lambda\right] + k(L)\left[(1-\lambda) - p_L(1-2\lambda)\right] \tag{26}$$

where $p_L \equiv \frac{\lambda p(\widehat{a})}{\lambda p(\widehat{a}) + (1-\lambda)(1-p(\widehat{a}))}$

Note that, since we have a continuum of entrepreneurs, at equilibrium the investment levels of all those who have received the same signal must necessarily be identical. It is straightforward to verify that the solution to the system composed by (21), and (24) gives $k(H)$ and $k(L)$ that satisfy

$$k(H) - k(L) = \alpha(\widehat{a})\left[E(\omega \mid H) - E(\omega \mid L)\right] \tag{27}$$

where $\alpha(\widehat{a}) \equiv \frac{[p(\widehat{a})(2\lambda-1)+1-\lambda][\lambda+p(\widehat{a})(1-2\lambda)]}{p(\widehat{a})(2\lambda-1)^2(1-\theta)(1-p(\widehat{a}))+\lambda(1-\lambda)}$. Note that

$$\alpha(\widehat{a}) - \frac{1}{1-\theta} = \frac{\theta}{\theta-1}\frac{\lambda(1-\lambda)}{\lambda(1-\lambda) + p(\widehat{a})\left(1-p(\widehat{a})\right)(2\lambda-1)^2(1-\theta)}. \tag{28}$$

Given the restriction $\theta < 1$, the sign of the above expression is the opposite sign to that of θ. This proves that, when $\theta > 0$, the responsiveness of investment to signal realization is greater under centralization, while the opposite holds when $\theta < 0$. ■

Proof of Proposition 1:

The Proof consists in two parts. First, we show that the optimal amount of investment by the ruler is uniquely defined in both the centralized and decentralized cases. We then show that, for $\theta > 0$, the ruler's equilibrium level of investment is strictly greater under centralization, while the opposite occurs when $\theta < 0$.

Equilibrium investment by the ruler

Centralized Information

Taking the entrepreneurs' beliefs over a as given, the ruler maximizes

$$p(a)\left((1-\lambda)E(K \mid H) + \lambda E(K \mid L)\right) + (1-p(a))\left((1-\lambda)E(K \mid L) + \lambda E(K \mid H)\right) - C(a). \quad (29)$$

From (19), $E(K \mid s) = E(k \mid s)$ for $s = H, L$. Hence, at equilibrium, the level of resources invested by the ruler satisfies:

$$(1-2\lambda)p'(a)\left(E(k \mid H) - E(k \mid L)\right) - C'(a) = 0. \quad (30)$$

Substituting for $E(k \mid s) = \frac{E(\omega|s)}{1-\theta}$, $s = H, L$, (30) becomes

$$\frac{1-2\lambda}{1-\theta}p'(a)\left[E(\omega \mid H) - E(\omega \mid L)\right] - C'(a) = 0. \quad (31)$$

Substituting for $E(\omega \mid H) - E(\omega \mid L)$ in (31) we obtain:

$$\left(\frac{p(1-p)}{1-\theta}\right)\left(\frac{(1-2\lambda)^2(h-l)}{\lambda(1-\lambda) + p(1-p)(1-2\lambda)^2}\right)p'(a) - C'(a) = 0. \quad (32)$$

Given our assumptions on $p(.)$ and $C(.)$ and θ it is straightforward to show that, evaluated at $a = 0$, the l.h.s. of (32) is strictly positive. Evaluated at $p(a) = 1$, the l.h.s. of (32) is strictly negative. Hence, (32) has at least one interior solution. We now show that the solution to (32) is in fact unique. The derivative of the l.h.s. of (32) with respect to a is

$$\eta\frac{\lambda(1-\lambda)(h-l)(1-2\lambda)^2(p'(a))^2}{1-\theta} + \left(\frac{\frac{p(1-p)}{1-\theta}(1-2\lambda)^2(h-l)}{\lambda(1-\lambda) + p(1-p)(1-2\lambda)^2}\right)p''(a) - C''(a) \quad (33)$$

where

$$\eta = \frac{1-2p}{(\lambda + p\,(1-2\lambda))^2\,(1-\lambda - p\,(1-2\lambda))^2}. \tag{34}$$

For $p \geq 1/2$, η is ≤ 0, so (33) is strictly negative. For $p < 0.5$, η is positive, but strictly decreasing in p. Hence, there exist a unique value of a that satisfies (32).

Decentralized Information

Taking the entrepreneurs' beliefs as given, the ruler maximizes

$$p(a)\,((1-\lambda)\,k(H) + \lambda k(L)) + (1-p(a))\,((1-\lambda)\,k(L) + \lambda k(H)) - C(a). \tag{35}$$

At equilibrium, the amount of resources invested by the ruler satisfies

$$(1-2\lambda)\,p'(a)\,(k(H) - k(L)) - C'(a) = 0. \tag{36}$$

Substituting for $k(H) - k(L)$ from (27) (36) becomes

$$\alpha(a)\,(1-2\lambda)\,p'(a)\,[E\,(\omega \mid H) - E\,(\omega \mid L)] - C'(a) = 0 \tag{37}$$

where $\alpha(a) = \frac{[p(a)(2\lambda-1)+1-\lambda][\lambda+p(a)(1-2\lambda)]}{p(a)(2\lambda-1)^2(1-\theta)(1-p(a))+\lambda(1-\lambda)}$ (since at equilibrium the entrepreneurs' conjecture must be correct). Substituting for $E\,(\omega \mid H) - E\,(\omega \mid L)$ in (37) we obtain:

$$\alpha(a)\varphi p'(a) - C'(a) = 0 \tag{38}$$

where $\varphi \equiv \frac{(2\lambda-1)^2 p(a)(1-p(a))(h-l)}{[\lambda+p(a)(1-2\lambda)][(1-p(a))(1-\lambda)+\lambda p(a)]}$. Given our assumptions on $p(.)$ and $C(.)$ it is straightforward to show that, evaluated at $a = 0$, the l.h.s. of (38) is strictly positive. Evaluated at $p(a) = 1$, the l.h.s. of (38) is strictly negative. Hence, (38) has at least one interior solution. We now show that the solution to (38) is in fact unique. The derivative of the l.h.s. of (38) with respect to a is

$$\Lambda p'(a)^2 + \alpha(a)\varphi p''(a) - C''(a) \tag{39}$$

where $\Lambda \equiv \frac{\lambda(1-\lambda)(h-l)(2\lambda-1)^2}{\left[\lambda(1-\lambda)+p(a)(1-p(a))(2\lambda-1)^2(1-\theta)\right]^2}(1-2p(a))$. For $p(a) \geq 1/2$, Λ is ≤ 0, so (39) is strictly negative. For $p(a) < 0.5$, Λ is positive, but strictly decreasing in a. Hence, there exist a unique value of a that satisfies (38).

Comparison between ruler's investment choice under centralization and under decentralization

Denote the equilibrium levels of resource investment under decentralization as a^D, and that under centralization as a^C. Consider the l.h.s. of (38), evaluated at a^C. This gives

$$(1-2\lambda)\, p'(a) \left(\alpha(a) - \frac{1}{1-\theta}\right). \tag{40}$$

As shown in the proof of lemma 1, the sign of $\alpha(a) - \frac{1}{1-\theta}$ is the inverse of that of θ. Hence, (40) is negative for $\theta > 0$, and positive for $\theta < 0$. We conclude that $a^D < a^C$ for $\theta > 0$, and $a^D > a^C$ for $\theta < 0$.■

Proof of Proposition 2: As the total mass of entrepreneurs is one, the mass of individuals in each group is equal to $\frac{1}{n}$. Consider an individual who has observed a signal with realization s, $s = H, L$. The individual's investment choice is given by

$$k(s) = \theta E(K \mid s) + E(\omega \mid s) \tag{41}$$

We have:

$$E(K|H) = \frac{k(H)}{n} + \frac{p_H}{n} \sum_{x=0}^{n-1} \left((xk(H) + (n-1-p)k(L)) \binom{n-1}{x} (1-\lambda)^x \lambda^{n-1-x} \right) + \tag{42}$$
$$\frac{1-p_H}{n} \sum_{x=0}^{n-1} \left((xk(H) + (n-1-x)k(L)) \binom{n-1}{x} \lambda^x (1-\lambda)^{n-1-x} \right)$$

where $p_H \equiv \frac{(1-\lambda)p(\widehat{a})}{(1-\lambda)p(\widehat{a})+\lambda(1-p(\widehat{a}))}$ ($\widehat{a}$ indicates the entrepreneurs' conjecture about a). Similarly,

$$E(K|L) = \frac{k(L)}{n} + \frac{p_L}{n}\sum_{x=0}^{n-1}\left((xk(H)+(n-1-x)k(L))\binom{n-1}{x}(1-\lambda)^x\lambda^{n-1-x}\right)+ \\ \frac{1-p_L}{n}\sum_{x=0}^{n-1}\left((xk(H)+(n-1-x)k(L))\binom{n-1}{x}\lambda^x(1-\lambda)^{n-1-x}\right) \tag{43}$$

where $p_L \equiv \frac{\lambda p(\widehat{a})}{\lambda p(\widehat{a})+(1-\lambda)(1-p(\widehat{a}))}$. It is straightforward to show that the (42) and (43) simplify to

$$\begin{array}{l} E(K|H) = \frac{k(H)}{n} + \frac{p_H}{n}(n-1)\left[k(H)(1-\lambda)+\lambda k(L)\right] + \frac{1-p_H}{n}(n-1)\left[k(L)(1-\lambda)+\lambda k(H)\right] \\ E(K \mid L) = \frac{k(L)}{n} + \frac{p_L}{n}(n-1)\left[k(H)(1-\lambda)+\lambda k(L)\right] + \frac{1-p_L}{n}(n-1)\left[k(L)(1-\lambda)+\lambda k(H)\right] \end{array} \tag{44}$$

The values of $k(H)$ and $k(L)$ that satisfy the system composed by (41) and (44) yield

$$k(H)-k(L) = n\frac{\left[1-\lambda-p(a)(1-2\lambda)\right]\left[p(a)(1-2\lambda)+\lambda\right]\left[E(\omega \mid H)-E(\omega \mid L)\right]}{\lambda(n-\theta)(1-\lambda)+p(a)(1-p(a))n(2\lambda-1)^2(1-\theta)}. \tag{45}$$

Taking the entrepreneurs' beliefs as given, the ruler maximizes

$$\frac{p(a)}{n}\sum_{x=0}^{n}\left((xk(H)+(n-x)k(L))\binom{n}{x}(1-\lambda)^x\lambda^{n-x}\right)+ \\ \frac{1-p(a)}{n}\sum_{x=0}^{n}\left((xk(H)+(n-x)k(L))\binom{n}{x}\lambda^x(1-\lambda)^{n-x}\right) - C(a). \tag{46}$$

It is straightforward to show that (46) simplifies to

$$p(a)\left[k(H)(1-\lambda)+k(L)\lambda\right]+(1-p(a))\left[k(H)\lambda-k(L)(1-\lambda)\right]-C(a) \tag{47}$$

The equilibrium level of investment by the ruler satisfies

$$p'(a)\left(k(H)-k(L)\right)(1-2\lambda)-C'(a)=0 \tag{48}$$

As seen in the proof of proposition 1, the value of $k(H)-k(L)$ determines the strenght of the ruler's incentives. Hence, to see how the ruler's incentives vary with n, we need to calculate the effect of changing n on $k(H)-k(L)$. Straightforward computations show that

$$\frac{d\left(k(H)-k(L)\right)}{dn} = -\left(1-\lambda\right)\theta\lambda\frac{\left[1-\lambda-p(a)\left(1-2\lambda\right)\right]\left[p(a)\left(1-2\lambda\right)+\lambda\right]\left[E(\omega\mid H)-E(\omega\mid L)\right]}{\left(\lambda\left(n-\theta\right)\left(1-\lambda\right)+p(a)(1-p)n\left(2\lambda-1\right)^2\left(1-\theta\right)\right)^2}. \tag{49}$$

With strategic complementarities ($\theta>0$), the ruler's incentives to invest are strictly decreasing in n, while under substitutability ($\theta<0$) they are strictly increasing in n. Notice that, evaluated at $n=1$, $\frac{d(k(H)-k(L))}{dn} = -\theta\frac{\lambda}{(\theta-1)^2}\frac{1-\lambda}{[p(a)(1-2\lambda)+\lambda]^2[1-\lambda-p(a)(1-2\lambda)]^2}$. Hence, for $\theta\to 1$, there is an infinitely large difference between the incentives provided when $n=1$, and those provided when $n=2$. ■

Proof of Proposition 3: First, we show that, with complementarity, at equilibrium $k(H)-k(L)$ is greater under centralization, while the opposite holds with substitutability. Second, we show that this ensures that the unique equilibrium of the game displays the properties described in Proposition 1.

Denote equilibrium investment levels under decentralization and centralization as $k^{\mathbf{D}}$ and $k^{\mathbf{C}}$, respectively. Under centralization, $E\left(K\mid H\right)=k^{\mathbf{C}}(H)$ and $E\left(K\mid L\right)=k^{\mathbf{C}}(L)$, so the optimal investment is

$$k^{\mathbf{C}}(H) = f(k^{\mathbf{C}}(H), E(\omega\mid H))$$
$$k^{\mathbf{C}}(L) = f(k^{\mathbf{C}}(L), E(\omega\mid L)).$$

Under decentralization, $E\left(K\mid H\right)=k^{\mathbf{D}}(H)\left[p_H(1-2\lambda)+\lambda\right]+k^{\mathbf{D}}(L)\left[(1-\lambda)-p_H(1-2\lambda)\right]$ and $E\left(K\mid L\right)=k^{\mathbf{D}}(H)\left[p_L(1-2\lambda)+\lambda\right]+k^{\mathbf{D}}(L)\left[(1-\lambda)-p_L(1-2\lambda)\right]$ so

$$k^{\mathbf{D}}(H) = f\left(k^{\mathbf{D}}(H)\left[p_H(1-2\lambda)+\lambda\right]+k^{\mathbf{D}}(L)\left[(1-\lambda)-p_H(1-2\lambda)\right],\ E(\omega\mid H)\right) \tag{50}$$
$$k^{\mathbf{D}}(L) = f\left(k^{\mathbf{D}}(H)\left[p_L(1-2\lambda)+\lambda\right]+k^{\mathbf{D}}(L)\left[(1-\lambda)-p_L(1-2\lambda)\right],\ E(\omega\mid L)\right) \tag{51}$$

where $p_H \equiv \frac{(1-\lambda)p(\widehat{a})}{(1-\lambda)p(\widehat{a})+\lambda(1-p(\widehat{a}))}$ and $p_L \equiv \frac{\lambda p(\widehat{a})}{\lambda p(\widehat{a})+(1-\lambda)(1-p(\widehat{a}))}$. Note that $E(K \mid H)$ can be rewritten as

$$k^{\mathbf{D}}(H) - \left(k^{\mathbf{D}}(H) - k^{\mathbf{D}}(L)\right)(p_H(2\lambda - 1) + 1 - \lambda) \tag{52}$$

and $E(K \mid L)$ can be rewritten as

$$k^{\mathbf{D}}(L) + \left(k^{\mathbf{D}}(H) - k^{\mathbf{D}}(L)\right)(\lambda + p_L(1 - 2\lambda)). \tag{53}$$

Hence, overall, we can write

$$\begin{aligned} k^{\mathbf{D}}(H) &= f(k^{\mathbf{D}}(H) - \Delta_H,\ E(\omega \mid H)) \\ k^{\mathbf{C}}(H) &= f(k^{\mathbf{C}}(H),\ E(\omega \mid H)) \\ k^{\mathbf{D}}(L) &= f(k^{\mathbf{D}}(L) + \Delta_L,\ E(\omega \mid L)) \\ k^{\mathbf{C}}(L) &= f(k^{\mathbf{C}}(L),\ E(\omega \mid L)) \end{aligned} \tag{54}$$

where $\Delta_H \equiv \left(k^{\mathbf{D}}(H) - k^{\mathbf{D}}(L)\right)((1-\lambda)(1-p_H) + p_H\lambda)$ and $\Delta_L \equiv \left(k^{\mathbf{D}}(H) - k^{\mathbf{D}}(L)\right)(\lambda + p_L(1-2\lambda))$. There are two cases to consider.

(i) $k^{\mathbf{D}}(H) > k^{\mathbf{D}}(L)$

In this case, both Δ_H and Δ_L are > 0. Suppose that $f(.)$ is strictly increasing in $E(K)$, i.e. there are strategic complementarities in investment. From (54), under centralization, the entrepreneurs' best reply function $f(.)$ when $\varsigma = H$ is strictly above the best reply function under decentralization. This implies that $k^{\mathbf{C}}(H) > k^{\mathbf{D}}(H)$. To see why, note that under A1, the best reply function $f(.)$ must cross the 45 degrees line from above. So $f(k - \Delta_H, E(\omega \mid H)) - k > 0$ for $k < k^{\mathbf{D}}(H)$ and < 0 for $k > k^{\mathbf{D}}(H)$. Now, $k^{\mathbf{C}}(H)$ solves $f(k, E(\omega \mid H)) - k = 0$. Consider $f(k - \Delta_H, E(\omega \mid H)) - k$. Evaluated at $k = k^{\mathbf{C}}(H)$, this gives $f(k^{\mathbf{C}}(H) - \Delta_H, E(\omega \mid H)) - f(k^{\mathbf{C}}(H), E(\omega \mid H)) < 0$. Hence, $k^{\mathbf{C}}(H) > k^{\mathbf{D}}(H)$.

The opposite holds for the entrepreneurs' reaction function conditional on $\varsigma = L$. Under centralization, this is strictly below that under decentralization. Hence, following the same reasoning as above, we must have $k^{\mathbf{D}}(L) > k^{\mathbf{C}}(L)$. Overall, therefore, we conclude that with strategic complementarities, we have $k^{\mathbf{D}}(H) - k^{\mathbf{D}}(L) < k^{\mathbf{C}}(H) - k^{\mathbf{C}}(L)$. It is straightforward to show that the opposite holds with substitutability.

(ii) $k^{\mathbf{D}}(H) \leq k^{\mathbf{D}}(L)$

This case may emerge only when investments are strategic complements. Since $k^{\mathbf{D}}(H) \leq k^{\mathbf{D}}(L)$ both Δ_H and Δ_L are ≤ 0. Suppose first that $k^{\mathbf{D}}(H) < k^{\mathbf{D}}(L)$, so Δ_H and Δ_L are < 0. This implies that the under decentralization the best reply function $f(.)$ when $\varsigma = H$ is strictly above the best reply function under centralization, so that, following a similar argument as above, $k^{\mathbf{D}}(H) > k^{\mathbf{C}}(H)$. Similarly, we must have $k^{\mathbf{D}}(L) < k^{\mathbf{C}}(L)$. Now, it is straightforward to see that in the centralized case $k^{\mathbf{C}}(H) > k^{\mathbf{C}}(L)$ (since $f(k, E(\omega \mid H)) > f(k, E(\omega \mid L))$ for all k's). This however implies that $k^{\mathbf{D}}(H) > k^{\mathbf{C}}(H) > k^{\mathbf{C}}(L) > k^{\mathbf{D}}(L)$, a contradiction,.

Now suppose that $k^{\mathbf{D}}(H) = k^{\mathbf{D}}(L)$, so Δ_H and Δ_L are $= 0$. This implies that $k^{\mathbf{D}}(H) = k^{\mathbf{C}}(H)$ and $k^{\mathbf{D}}(L) = k^{\mathbf{C}}(L)$. Since $k^{\mathbf{C}}(H) > k^{\mathbf{C}}(L)$, this again entails a contradiction. This proves that case (ii) may never emerge, and the only relevant case is (i).

Now consider the ruler's incentives. From the proof of Proposition 1, we know that at equilibrium we must have

$$(1 - 2\lambda)\, p'(a)\, (k(H, p(a)) - k(L, p(a))) - C'(a) = 0 \tag{55}$$

where $k(\varsigma, p(a))$ denotes the equilibrium level of investment of an entrepreneur who has observed a signal $\varsigma = H, L$ and correctly believes that the probability with which $\omega = h$ is equal to $p(a)$. Given our assumptions on $p(.)$ and $C(.)$ and θ it is straightforward to show that (55) has at least one interior solution: evaluated at $a = 0$, the l.h.s. of (55) is strictly positive; evaluated at $p(a) = 1$, the l.h.s. of (55) is strictly negative. Assumption A3 is sufficient to ensure that (55) has a unique solution.

Finally, note that (55) can be rewritten as

$$k(H, p(a)) - k(L, p(a)) = \frac{C'(a)}{p'(a)(1 - 2\lambda)}. \tag{56}$$

The r.h.s. of (56) is strictly increasing in a. Suppose that $\theta > 0$ (the case where $\theta < 0$ is analogous). As shown above, for any value of a we then have $k^{\mathbf{C}}(H, p(a)) - k^{\mathbf{C}}(L, p(a)) > k^{\mathbf{D}}(H, p(a)) - k^{\mathbf{D}}(L, p(a))$. Hence, we must necessarily have $a^C > a^D$. ■